Surigao City Campus

Bachelor of Science in Electrical Engineering

SURIGAO STATE COLLEGE OF TECHNOLOGY

SSCT

"For Nation's Greater Heights"

I.7.7. learning modules;

Module no. 1
DC Circuits

Topic: 1.1 Basic Concepts
1.2 Basic Circuit Laws
1.3 Analysis Methods
1.4 Circuit Analysis Techniques
1.5 Capacitors and Inductors
1.6 First-Order Circuits
1.7 Second-Order Circuits

Time Frame: 7 hrs.

Introduction:
This module covers the overall topics of DC circuits which fundamentally emphasizes the electric circuit theory as an essential course for electrical engineering students. In this module, the basic concepts, fundamental circuit laws and other important theories and concepts in dc circuits are concisely discussed.

Objectives:
At the end of this module, the student shall be able to

1. Recall the fundamental electrical concepts, laws, and theories in dc circuits and
2. Solve various engineering problems in dc circuits using the concepts and, laws and theories reviewed.

Pre – Test

Module 1 – DC Circuits

Name: **Subject:**
Course/Section: **Date:**

Direction: Read the questions carefully. Write your answers in a separate sheet of paper.

1. What are the circuits which are basically known as first-order circuits?
2. What is/are the differences between capacitors and inductors when in dc?
3. Why is RLC circuit called as second-order circuit?
4. What circuit laws forms the basis for nodal and mesh analysis?
5. What is the necessary circuit condition for maximum power transfer?

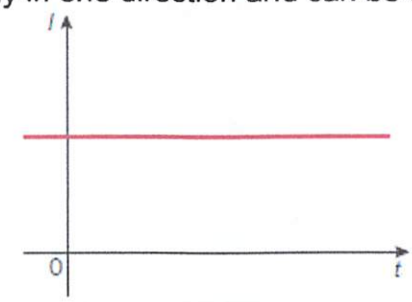
Learning Activities:
1.1 BASIC CONCEPTS
Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).

$$q = it$$

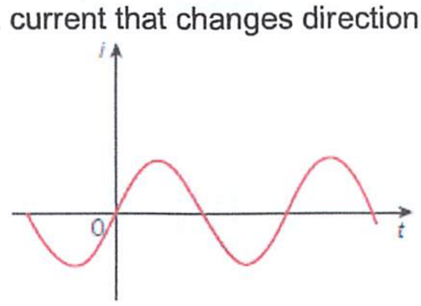
Electric current is the time rate of change of charge, measured in amperes (A).

$$i = \frac{dq}{dt}$$

where current is measured in amperes (A), and 1 ampere = 1 coulomb/second.
Direct current (dc) flows only in one direction and can be constant or time varying.



Alternating current (ac) is a current that changes direction with respect to time.



Voltage (or **potential difference**) is the energy required to move a unit charge from a reference point (−) to another point (+), measured in volts (V).

$$v = \frac{dw}{dq}$$

where *w* is energy in joules (J) and *q* is charge in coulombs (C). The voltage *v* is measured in volts (V), named in honor of the Italian physicist Alessandro Antonio Volta (1745-1827), who invented the first voltaic battery.
Power is the time rate of expending or absorbing energy, measured in watts (W).

$$p = \frac{dw}{dt} = vi$$

where *p* is power in watts (W), *w* is energy in joules (J), and *t* is time in seconds (s). It is also the product between voltage and current.
Passive sign convention is satisfied when the current enters through the positive terminal of an element and p = +vi. If the current enters through the negative terminal, p = −vi.
By **law of conservation of energy** the algebraic sum of power in a circuit at any instant

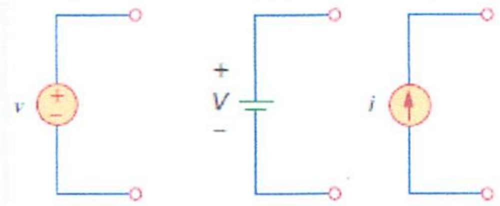
of time is zero.

$$\Sigma p = 0$$

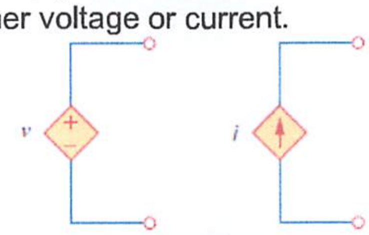
Energy is the capacity to do work, measured in joules (J).

$$1 \text{ Wh} = 3600 \text{ J}$$

Ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.



Ideal dependent (or **controlled**) **source** is an active element in which the source quantity is controlled by another voltage or current.



The four possible types of dependent sources are:
1. A voltage-controlled voltage source (VCVS).
2. A current-controlled voltage source (CCVS).
3. A voltage-controlled current source (VCCS).
4. A current-controlled current source (CCCS).

Example 1.1
How much charge is represented by 4,600 electrons?

Solution:
Each electron has -1.602×10^{-19} C. Hence 4,600 electrons will have -1.602×10^{-19} C/electron $\times$ 4,600 electrons = -7.369×10^{-16} C

Example 1.2
An energy source forces a constant current of 2 A for 10 s to flow through a light bulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:
The total charge is

$$\Delta q = i\Delta t = 2 \times 10 = 20 \text{ C}$$

The voltage drop is

$$v = \frac{\Delta w}{\Delta q} = \frac{2.3 \times 10^3}{20} = 115 \text{ V}$$

Example 1.3
Find the power delivered to an element at t = 3 ms if the current entering its positive terminal is

$$i = 5\cos 60\pi t \text{ A}$$

and the voltage is: (a) $v = 3i$,

Solution:
(a) The voltage is $v = 3i = 15\cos 60\pi t$; hence, the power is

$$p = vi = 75\cos^2 60\pi t \text{ W}$$

At t = 3 ms,

$$p = 75\cos^2(60\pi \times 3 \times 10^{-3}) = 75\cos^2 0.18\pi = 53.48 \text{ W}$$

Example 1.4
How much energy does a 100-W electric bulb consume in two hours?

Solution:

$$w = pt = 100\text{ (W)} \times 2\text{ (h)} \times 60\text{ (min/h)} \times 60\text{ (s/min)}$$
$$= 720{,}000 \text{ J} = 720 \text{ kJ}$$

This is the same as

$$w = pt = 100 \text{ W} \times 2 \text{ h} = 200 \text{ Wh}$$

1.2 BASIC CIRCUIT LAWS
Resistance is the physical property or ability to resist current and is represented by the symbol R.

$$R = \rho\frac{\ell}{A}$$

where ρ is known as the *resistivity* of the material in ohm-meters, A is the cross-sectional area, and ℓ is length.
Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

$$v = iR$$

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

$$R = \frac{v}{i}$$

where 1 Ω = 1 V/A.
Short circuit is a circuit element with resistance approaching zero.
Open circuit is a circuit element with resistance approaching infinity.
Conductance is the ability of an element to conduct electric current; it is measured in mhos (℧) or siemens (S).

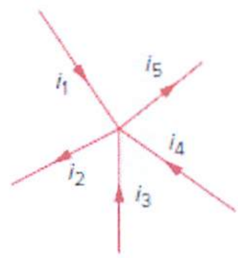
$$G = \frac{1}{R} = \frac{i}{v}$$

where 1 S = 1 ℧ = 1 A/V.
Branch represents a single element such as a voltage source or a resistor.

Node is the point of connection between two or more branches.
Loop is any closed path in a circuit.
Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero; or the sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.



The algebraic sum of currents at the node is

$$i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0$$

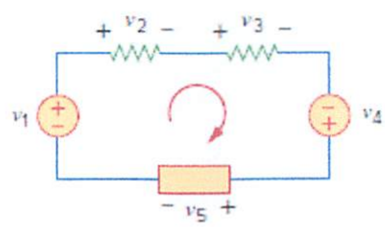
or

$$i_1 + i_3 + i_4 = i_2 + i_5$$

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero; or the sum of voltage drops = sum of voltage rises.

$$\sum_{m=1}^{M} v_m = 0$$

where M is the number of voltages in the loop (or the number of branches in the loop) and v_m is the mth voltage.



By KVL,

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

or

$$v_2 + v_3 + v_5 = v_1 + v_4$$

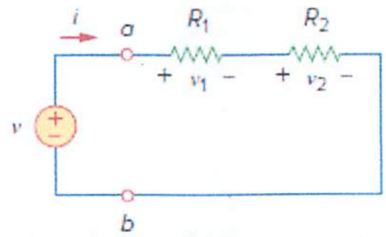
The **equivalent resistance** of any number of resistors connected in **series** is the sum of the individual resistances.

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

Principle of voltage division:

$$v_1 = \frac{R_1}{R_1 + R_2}v \quad v_2 = \frac{R_2}{R_1 + R_2}v$$

Voltage Divider Circuit:



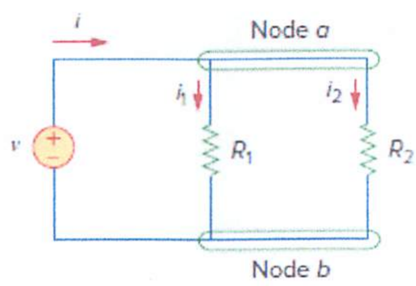
The **equivalent resistance** of two **parallel** resistors is equal to the product of their resistances divided by their sum.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Principle of Current Division:

$$i_1 = \frac{R_2}{R_1 + R_2}i \quad i_2 = \frac{R_1}{R_1 + R_2}i$$

Current Divider Circuit:



Delta to Wye Conversion:

Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three Δ resistors.

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

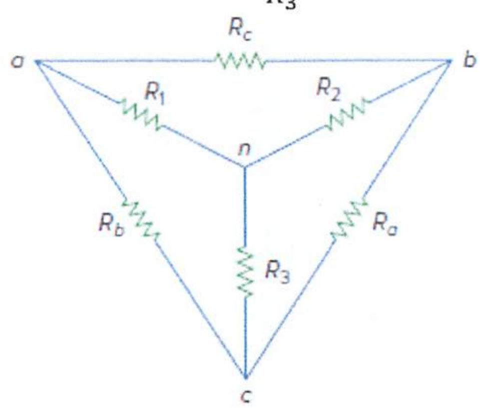
Wye to Delta Conversion:

Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$



When Y and Δ are balanced, then

$$R_1 = R_2 = R_3 = R_Y, \qquad R_a = R_b = R_c = R_\Delta$$

And the formulas become

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y$$

Example 1.5

A voltage source of 20 sin πt V is connected across a 5-kΩ resistor. Find the current through the resistor and the power dissipated.

Solution:

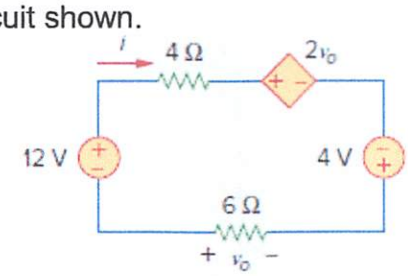
$$i = \frac{v}{R} = \frac{20\sin \pi t}{5 \times 10^3} = 4 \sin \pi t \text{ mA}$$

Hence,

$$p = vi = 80 \sin^2 \pi t \text{ mW}$$

Example 1.6

Determine v_o and i in the circuit shown.



Solution:

We apply KVL around the loop as shown in the next figure. The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad (1)$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad (2)$$

Substituting Eq. (2) into Eq. (1) yields

$$-16 + 10i - 12i = 0 \Rightarrow i = -8 \text{ A}$$

and $v_o = 48$ V.

Example 1.7
Find current i_o and voltage v_o in the circuit shown.

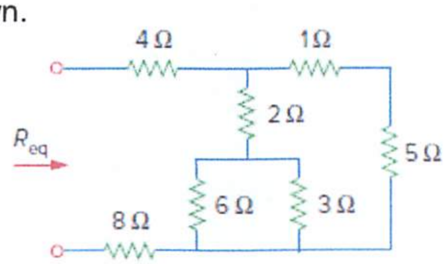
Solution:
Applying KCL to node *a*, we obtain

$$3 + 0.5i_o = i_o \Rightarrow i_o = 6 \text{ A}$$

For the 4-Ω resistor, Ohm's law gives

$$v_o = 4i_o = 24 \text{ V}$$

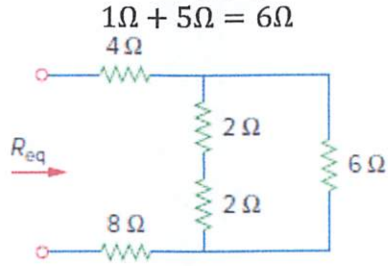
Example 1.8
Find R_{eq} for the circuit shown.



Solution:

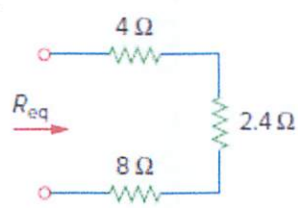
$$6\Omega \parallel 3\Omega = \frac{6 \times 3}{6+3} = 2\Omega$$

$$1\Omega + 5\Omega = 6\Omega$$



$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

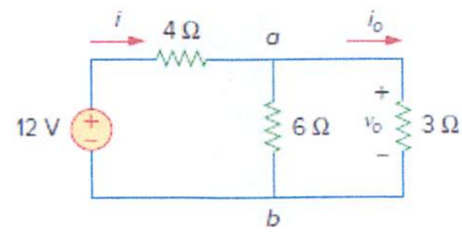
$$4\Omega \parallel 6\Omega = \frac{4 \times 6}{4+6} = 2.4\Omega$$



$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Example 1.9

Find i_o and v_o in the circuit shown below. Calculate the power dissipated in the 3-Ω resistor.

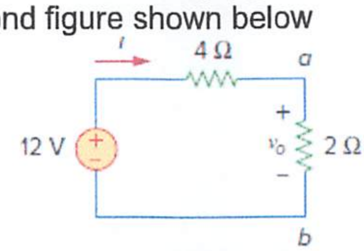


Solution:

The 6-Ω and 3-Ω resistors are in parallel

$$6\Omega \parallel 3\Omega = \frac{6\times 3}{6+3} = 2\Omega$$

our circuit reduces to the second figure shown below



There are two ways to obtain v_o. One way is to apply Ohm's law

$$i = \frac{12}{4+2} = 2\text{ A}$$

Hence, $v_o = 2i = 2 \times 2 = 4$ V

Another way is to apply voltage division,

$$v_o = \frac{2}{2+4}(12\text{ V}) = 4\text{ V}$$

Similarly, i_o can be obtained in two ways. First by Ohm's law in the first figure,

$$v_o = 3i_o = 4 \Rightarrow i_o = \frac{4}{3}\text{A}$$

Second is by current division,

$$i_o = \frac{6}{6+3}i = \frac{2}{3}(2\text{ A}) = \frac{4}{3}\text{A}$$

The power dissipated in the 3-Ω resistor is

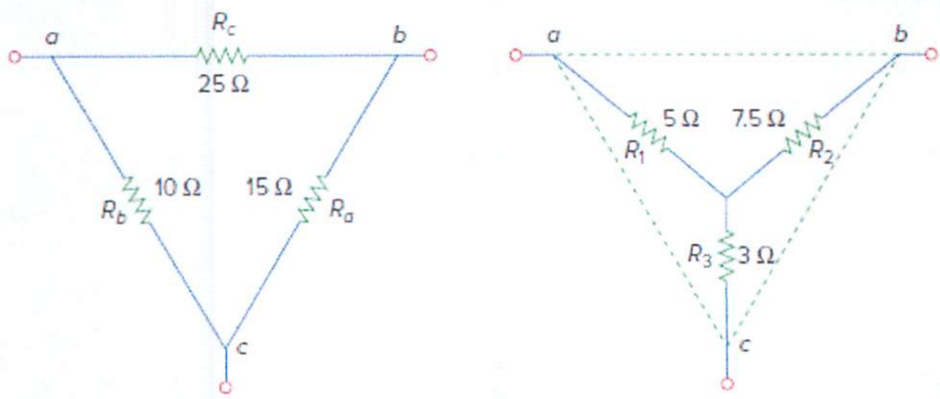
$$p_o = v_o i_o = 4\left(\frac{4}{3}\right) = 5.333\text{ W}$$

Example 1.10

Convert the Δ network to an equivalent Y network.

Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10\times 25}{15+10+25} = \frac{250}{50} = 5\ \Omega$$



$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

1.3 ANALYSIS METHODS

Nodal analysis is also known as the node-voltage method.

Steps to Determine Node Voltages:

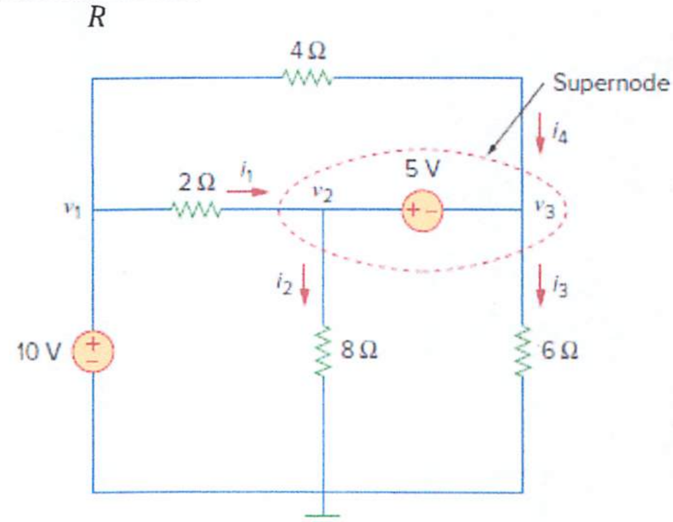
1. Select a node as the reference node. Assign voltages v_1, v_2, . . . , $v_n - 1$ to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

The number of nonreference nodes is equal to the number of independent equations.

When solving using nodal analysis, current flows from a higher potential to a lower potential in a resistor by the passive sign convention:

$$i = \frac{v_{\text{higher}} - v_{\text{lower}}}{R}$$

Supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.



CASE 1: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.

CASE 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a *generalized node* or *supernode*; we apply both KCL and KVL to determine the node voltages.

Important properties of a supernode:

1. The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
2. A supernode has no voltage of its own.
3. A supernode requires the application of both KCL and KVL.

Mesh analysis is also known as loop analysis or the mesh-current method. A **mesh** is a loop that does not contain any other loops within it.

Steps to Determine Mesh Currents:

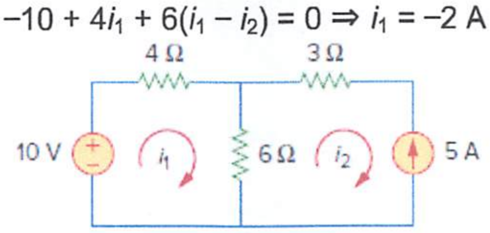
1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

The direction of the mesh current is arbitrary—(clockwise or counterclockwise)— and does not affect the validity of the solution.

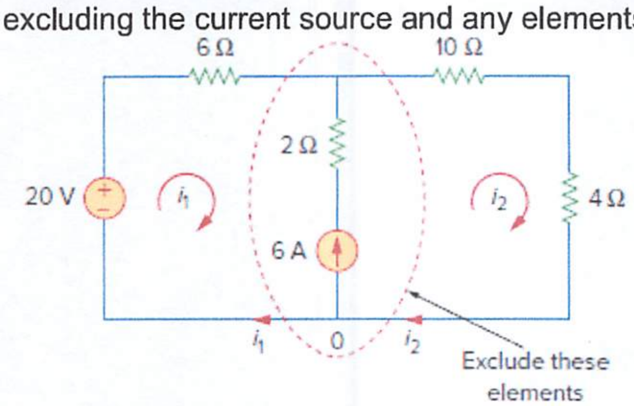
Supermesh results when two meshes have a (dependent or independent) current source in common.

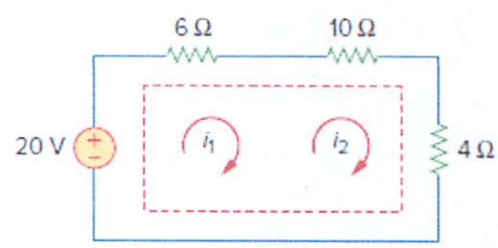
CASE 1 When a current source exists only in one mesh: Consider the circuit below, for example. We set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way; that is,

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2 \text{ A}$$



CASE 2 When a current source exists between two meshes: We create a *supermesh* by excluding the current source and any elements connected in series with it.



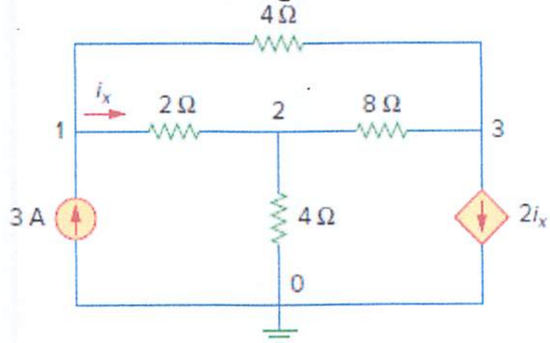


Important properties of a supermesh:

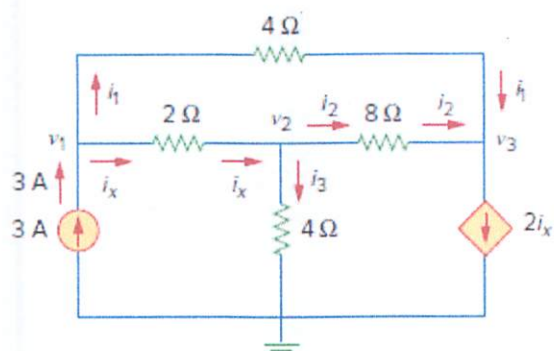
1. The current source in the supermesh provides the constraint equation necessary

to solve for the mesh currents.
2. A supermesh has no current of its own.
3. A supermesh requires the application of both KVL and KCL.

Example 1.11
Determine the voltages at the nodes in the figure shown below.



Solution:



At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3v_1 - 2v_2 - v_3 = 12 \qquad (1)$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$-4v_1 + 7v_2 - v_3 = 0 \qquad (2)$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$2v_1 - 3v_2 + v_3 = 0 \qquad (3)$$

Adding Eqs. (1) and (3).

$$5v_1 - 5v_2 = 12$$

$$v_1 - v_2 = \frac{12}{5} = 2.4 \qquad (4)$$

Adding Eqs. (2) and (3)

$$-2v_1 + 4v_2 = 0 \quad \Rightarrow \quad v_1 = 2v_2 \qquad (5)$$

Substituting Eq. (5) into Eq. (4)

$$2v_2 - v_2 = 2.4 \quad \Rightarrow \quad v_2 = 2.4, \quad v_1 = 2v_2 = 4.8 \text{ V}$$

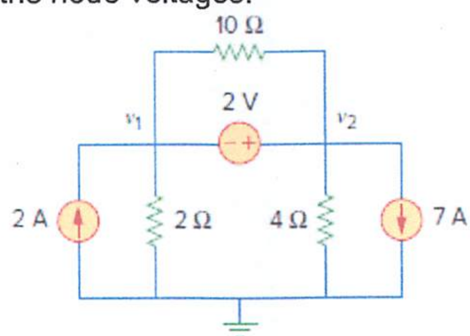
From Eq. (3)

$$v_3 = 3v_2 - 2v_1 = 3v_2 - 4v_2 = -v_2 = -2.4 \text{ V}$$

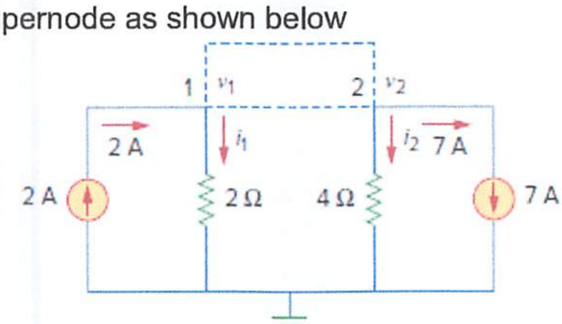
Thus,

$$v_1 = 4.8 \text{ V}, \quad v_2 = 2.4 \text{ V}, \quad v_3 = -2.4 \text{ V}$$

Example 1.12
For the circuit shown, find the node voltages.



Solution:
Applying KCL to the supernode as shown below



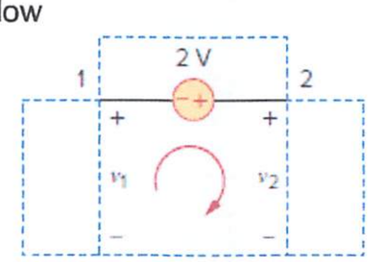
$$2 = i_1 + i_2 + 7$$

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

or

$$v_2 = -20 - 2v_1 \tag{1}$$

Applying KVL to the circuit below



$$-v_1 - 2 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 + 2 \tag{2}$$

From Eqs. (1) and (2),

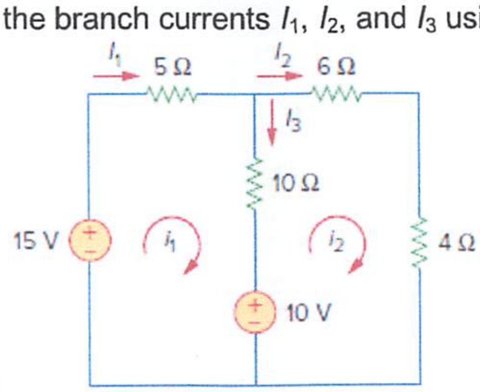
$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Rightarrow \quad v_1 = -7.333 \text{ V}$$

and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

Example 1.13
For the circuit shown, find the branch currents I_1, I_2, and I_3 using mesh analysis.



Solution:
For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad (1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \qquad (2)$$

Using Cramer's rule, we cast Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

We obtain the determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = 6 - 2 = 4$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \qquad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

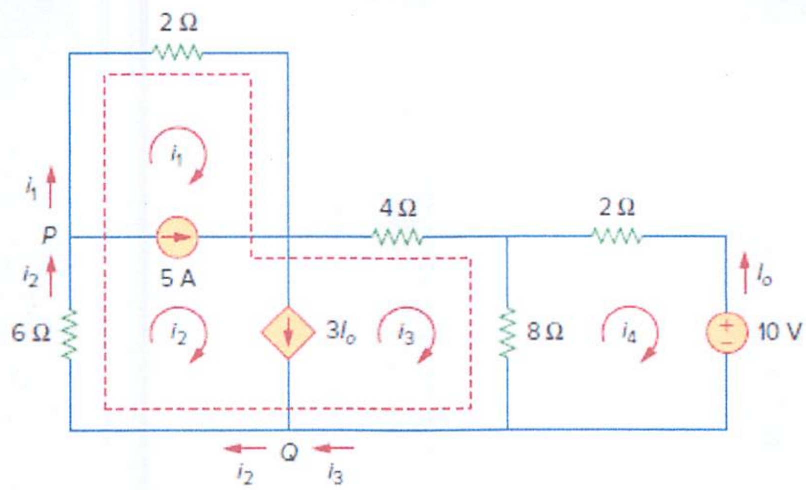
Thus,

$$i_1 = \frac{\Delta_1}{\Delta} = 1 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = 1 \text{ A}$$

$$i_3 = 0 \text{ A}$$

Example 1.14
For the circuit shown, find i_1 to i_4 using mesh analysis.

Solution:



Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \quad (1)$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \quad (2)$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$,

$$i_2 = i_3 - 3i_4 \quad (3)$$

In mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

$$5i_4 - 4i_3 = -5 \quad (4)$$

From Eqs. (1) to (4),

$$i_1 = -7.5 \text{ A}, \qquad i_2 = -2.5 \text{ A}, \qquad i_3 = 3.93 \text{ A}, \qquad i_4 = 2.143 \text{ A}$$

1.4 CIRCUIT ANALYSIS TECHNIQUES

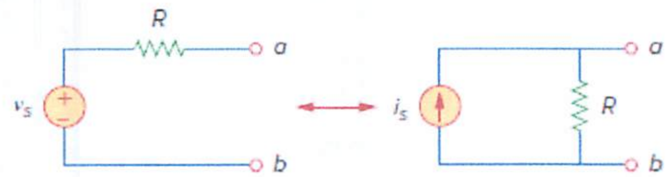
A **linear** circuit is one whose output is linearly related (or directly proportional) to its input.

Superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

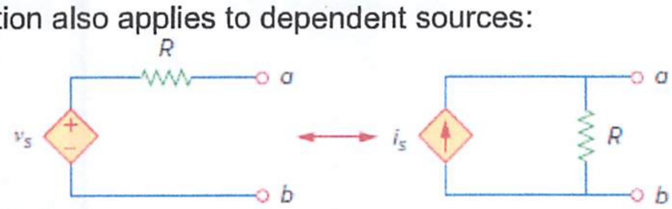
Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



Source transformation also applies to dependent sources:



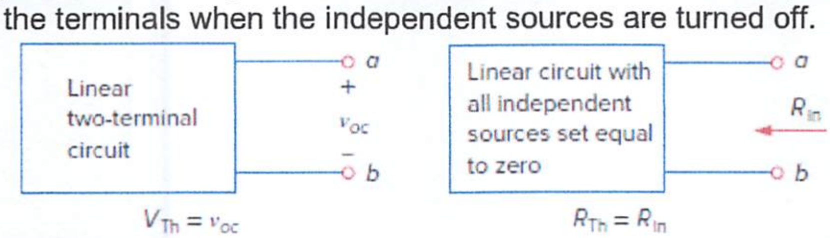
Source transformation requires that

$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Important points when dealing with source transformation:

1. Note from the figures above that the arrow of the current source is directed toward the positive terminal of the voltage source.
2. Note from the equation above that source transformation is not possible when $R = 0$, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, $R \neq 0$. Similarly, an ideal current source with $R = \infty$ cannot be replaced by a finite voltage source.

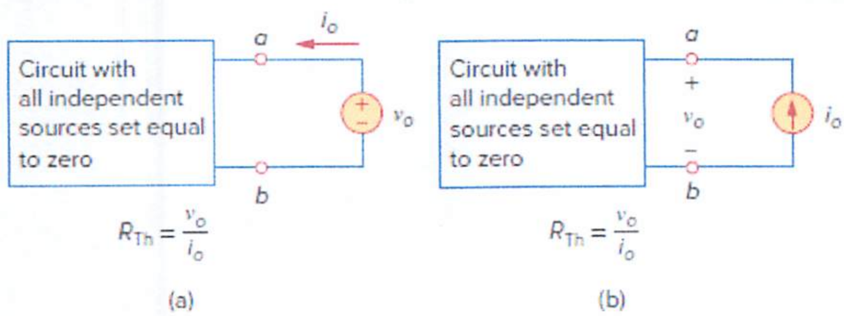
Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



Two cases when finding R_{Th}:

CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals *a* and *b*, as shown in Fig. (b).

CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals *a* and *b* and determine the resulting current i_o. Then $R_{Th} = v_o/i_o$, as shown in Fig. (a). Alternatively, we may insert a current source i_o at terminals *a-b* as shown in Fig. (b) and find the terminal voltage v_o. Again $R_{Th} = v_o/i_o$. Either of the two approaches will give the same result. In either approach we may assume any value of v_o and i_o.



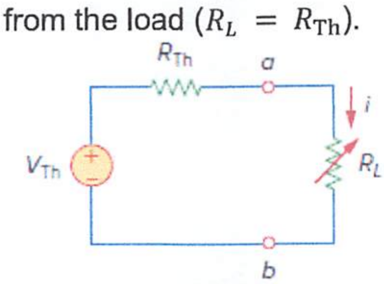
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.
By source transformation, the Thevenin and Norton resistances are equal:

$$R_N = R_{Th}$$

and

$$I_N = \frac{V_{Th}}{R_{Th}}$$

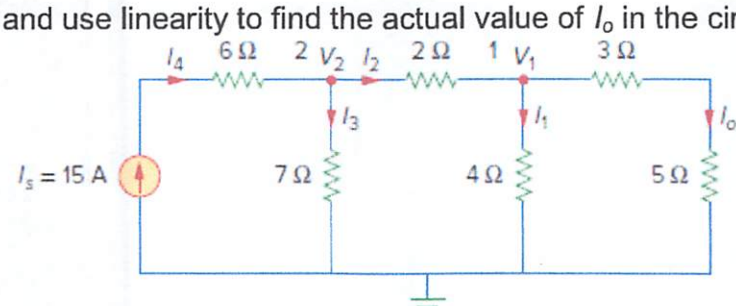
Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).



The maximum power transferred is obtained through:

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 1.15
Assume I_o = 1 A and use linearity to find the actual value of I_o in the circuit shown.



Solution:
If I_o = 1 A, then $V_1 = (3 + 5)I_o$ = 8 V and $I_1 = V_1/4$ = 2 A. Applying

KCL at node 1 gives

$$I_2 = I_1 + I_o = 3\text{ A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14\text{ V},\ I_3 = V_2/7 = 2\text{ A}$$

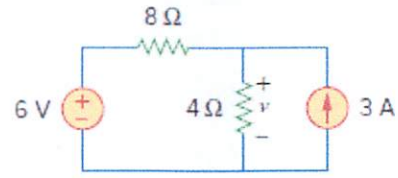
Applying KCL at node 2 gives

$$I_4 = I_3 + I_2 = 5\text{ A}$$

Therefore, $I_s = 5$ A. This shows that assuming $I_o = 1$ gives $I_s = 5$ A, the actual source current of 15 A will give $I_o = 3$ A as the actual value.

Example 1.16

Use the superposition theorem to find *v* in the circuit shown.



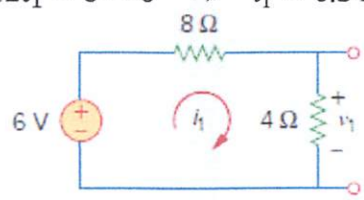
Solution:

Let

$$v = v_1 + v_2$$

Applying KVL to the loop gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5\text{ A}$$



Thus,

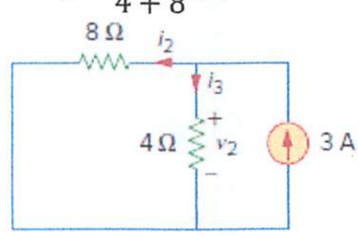
$$v_1 = 4i_1 = 2\text{ V}$$

Or

$$v_1 = \frac{4}{4+8}(6) = 2\text{ V}$$

To get v_2, we use current division,

$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$



Hence,

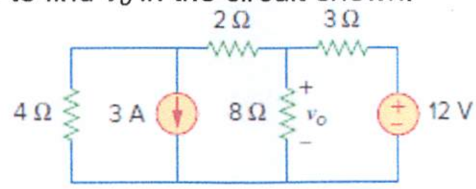
$$v_2 = 4i_3 = 8\text{ V}$$

And we find

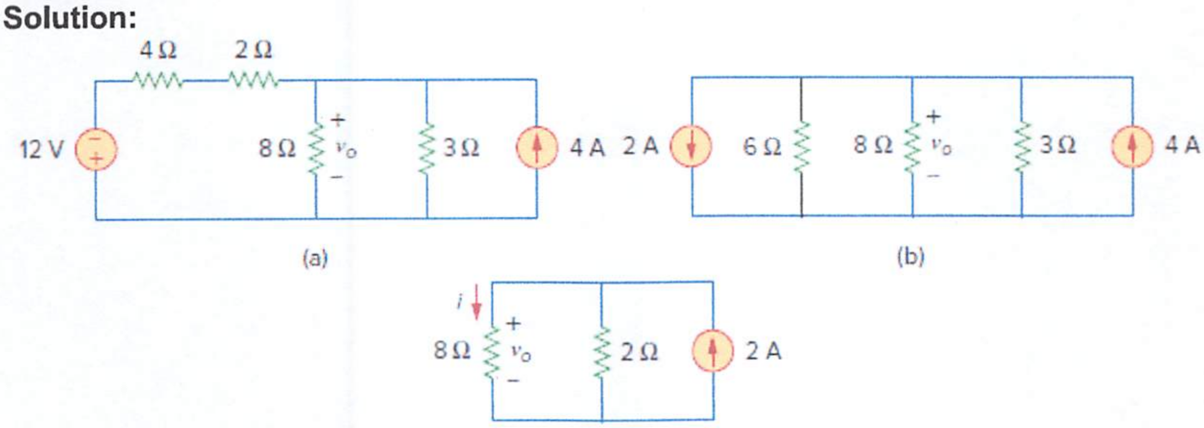
$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

Example 1.17

Use source transformation to find v_o in the circuit shown.



Solution:



(c)

We use current division in Fig. (c) to get

$$i = \frac{2}{2+8}(2) = 0.4 \text{ A}$$

and

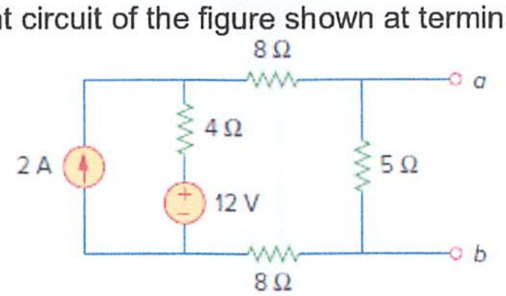
$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$

Alternatively, since the 8-Ω and 2-Ω resistors in Fig. 4.18(c) are in parallel, they have the same voltage v_o across them. Hence,

$$v_o = (8 \parallel 2)(2 \text{ A}) = \frac{8 \times 2}{10}(2) = 3.2 \text{ V}$$

Example 1.18

Find the Norton equivalent circuit of the figure shown at terminals *a-b*.



Solution:

$$R_N = 5 \parallel (8+4+8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\,\Omega$$

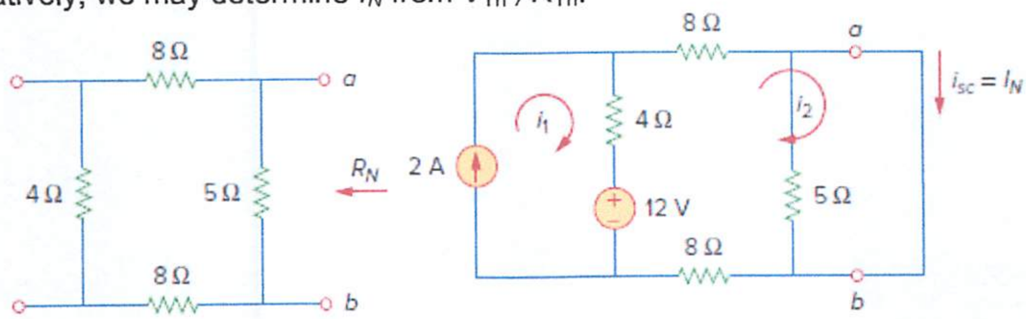
Applying mesh analysis

$$i_1 = 2 \text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

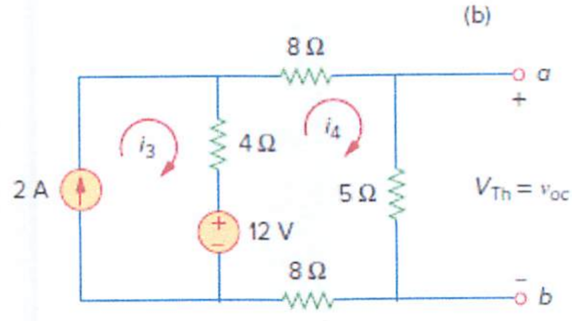
From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from V_{Th}/R_{Th}.



(a) (b)



(c)

Using mesh analysis, we obtain

$$i_3 = 2\text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow \quad i_4 = 0.8\text{ A}$$

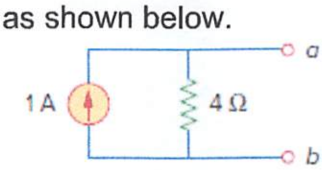
and

$$v_{oc} = V_{Th} = 5i_4 = 4\text{ V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1\text{ A}$$

The Norton equivalent circuit is as shown below.



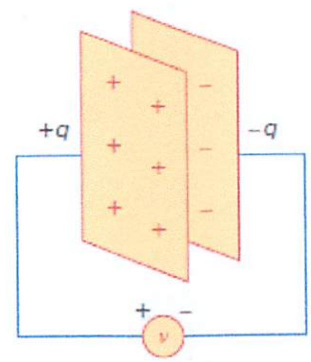
1.5 CAPACITORS AND INDUCTORS

A **capacitor** consists of two conducting plates separated by an insulator (or dielectric). The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly proportional to the applied voltage v so that

$$q = Cv$$

where C, the constant of proportionality, is known as the *capacitance* of the capacitor. The unit of capacitance is the farad (F), in honor of the English physicist Michael

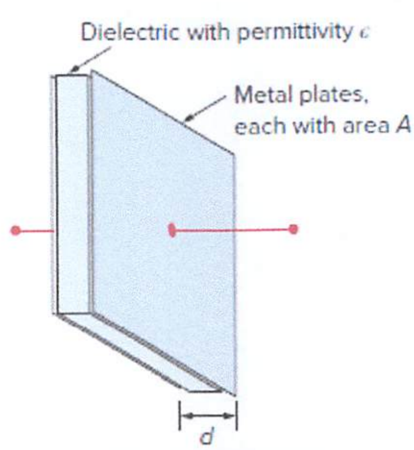
Faraday (1791–1867).



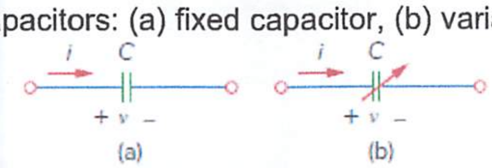
Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates, measured in farads (F).
For parallel-plate capacitor, the capacitance is given by

$$C = \frac{\epsilon A}{d}$$

where A is the surface area of each plate, d is the distance between the plates, and ϵ is the permittivity of the dielectric material between the plates.



The circuit symbols for capacitors: (a) fixed capacitor, (b) variable capacitor



Important properties of a capacitor:

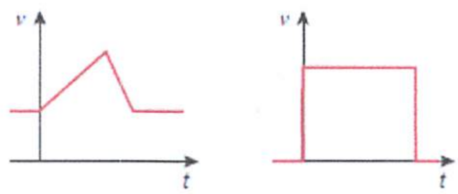
1. Note from the equation shown that when the voltage across a capacitor is not changing with time (i.e., dc voltage), the current through the capacitor is zero. Thus, **a capacitor is an open circuit to dc**. However, if a battery (dc voltage) is connected across a capacitor, the capacitor charges.

$$i = C\frac{dv}{dt}$$

2. The voltage on the capacitor must be continuous. The voltage on a capacitor

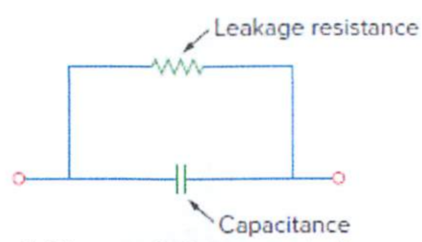
cannot change abruptly. The capacitor resists an abrupt change in the voltage across it. According to the equation above, a discontinuous change in voltage requires an infinite current, which is physically impossible. For example, the voltage across a capacitor may take the form shown in Fig. (a), whereas it is not physically possible for the capacitor voltage to take the form shown in Fig. (b) because of the abrupt changes. Conversely, the current through a capacitor can change instantaneously.

3.



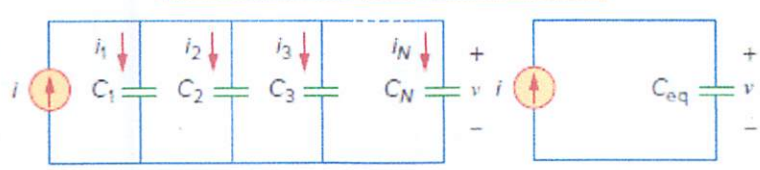
(a) (b)

4. The ideal capacitor does not dissipate energy. It takes power from the circuit when storing energy in its field and returns previously stored energy when delivering power to the circuit.
5. A real, nonideal capacitor has a parallel-model leakage resistance, as shown below. The leakage resistance may be as high as 100 M Ω and can be neglected for most practical applications.



The **equivalent capacitance** of *N* **parallel**-connected capacitors is the sum of the individual capacitances.

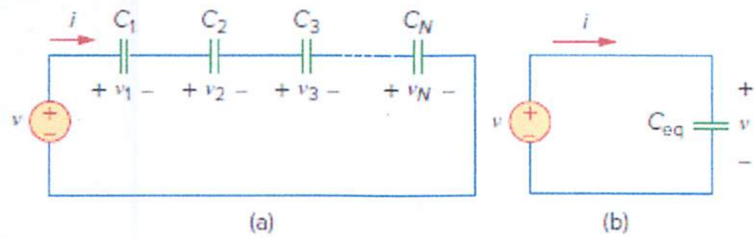
$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$



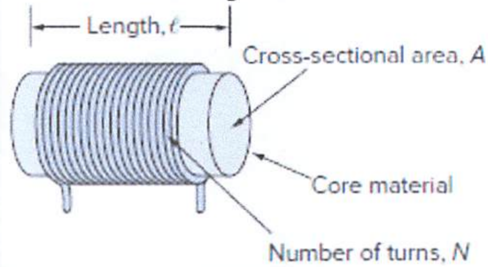
(a) (b)

The **equivalent capacitance** of **series**-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$



An **inductor** consists of a coil of conducting wire.

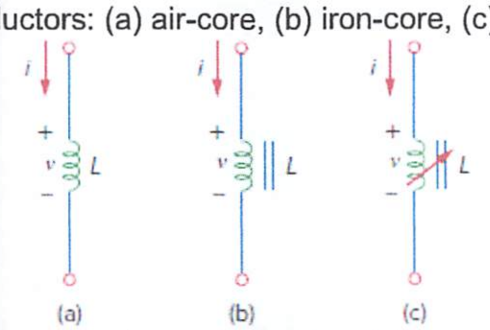


Using the passive sign convention the voltage across an inductor is

$$v = L\frac{di}{dt}$$

where *L* is the constant of proportionality called the *inductance* of the inductor. The unit of inductance is the henry (H), named in honor of the American inventor Joseph Henry (1797–1878).

The circuit symbols for inductors: (a) air-core, (b) iron-core, (c) variable iron-core



Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

The inductance of an inductor depends on its physical dimension and construction. For example, for the inductor, (solenoid) shown above,

$$L = \frac{N^2 \mu A}{\ell}$$

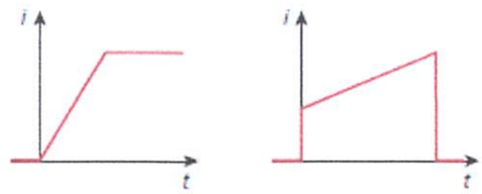
where *N* is the number of turns, ℓ is the length, A is the cross-sectional area, and *μ* is the permeability of the core.

Important properties of an inductor:

1. Note from the equation shown that the voltage across an inductor is zero when the current is constant. Thus, **an inductor acts like a short circuit to dc.**

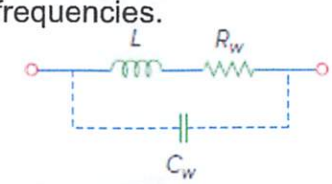
$$L = \frac{N^2 \mu A}{\ell}$$

2. An important property of the inductor is its opposition to the change in current flowing through it. The current through an inductor cannot change instantaneously. According to the equation above, a discontinuous change in the current through an inductor requires an infinite voltage, which is not physically possible. Thus, an inductor opposes an abrupt change in the current through it. For example, the current through an inductor may take the form shown in Fig. (a), whereas the inductor current cannot take the form shown in Fig. (b) in real-life situations due to the discontinuities. However, the voltage across an inductor can change abruptly.



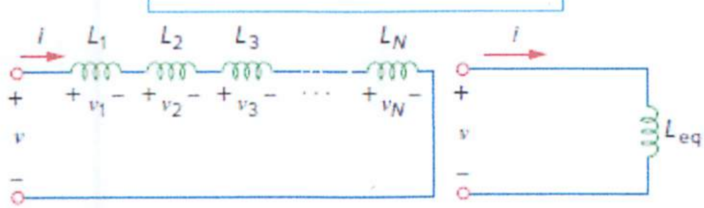
(a) (b)

3. Like the ideal capacitor, the ideal inductor does not dissipate energy. The energy stored in it can be retrieved at a later time. The inductor takes power from the circuit when storing energy and delivers power to the circuit when returning previously stored energy.
4. A practical, nonideal inductor has a significant resistive component, as shown below. This is due to the fact that the inductor is made of a conducting material such as copper, which has some resistance. This resistance is called the *winding resistance* R_w, and it appears in series with the inductance of the inductor. The presence of R_w makes it both an energy storage device and an energy dissipation device. Since R_w is usually very small, it is ignored in most cases. The nonideal inductor also has a *winding capacitance* C_w due to the capacitive coupling between the conducting coils. C_w is very small and can be ignored in most cases, except at high frequencies.



The **equivalent inductance** of **series**-connected inductors is the sum of the individual inductances.

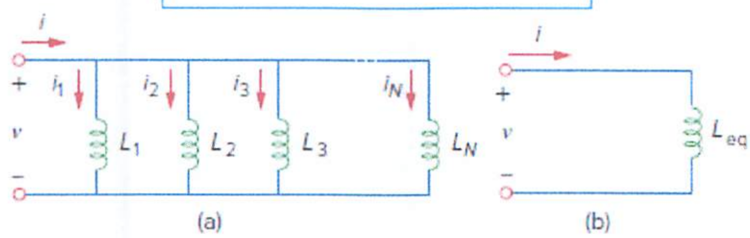
$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$



(a) (b)

The **equivalent inductance** of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

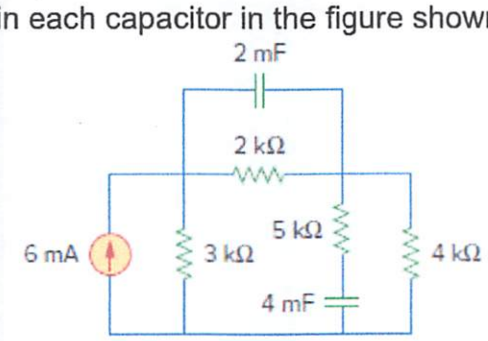


Important Characteristics of the Basic Elements

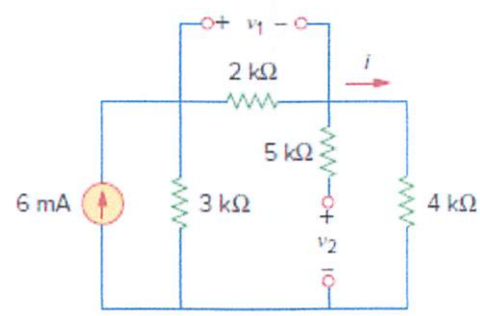
Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

Passive sign convention is assumed.

Example 1.19
Obtain the energy stored in each capacitor in the figure shown under dc conditions.



Solution:
Under dc conditions, we replace each capacitor with an open circuit, as shown.



The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

Hence,

$$v_1 = 2000i = 4\text{ V} \qquad v_2 = 4000i = 8\text{V}$$

The energies stored in the capacitors are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2\times10^{-3})(4^2) = 16\text{ mJ}$$
$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4\times10^{-3})(8^2) = 128\text{ mJ}$$

Example 1.20
Determine the voltage across a 2-μF capacitor if the current through it is

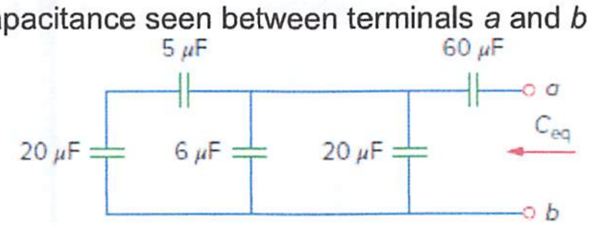
$$i(t) = 6e^{-3000t}\text{mA}$$

Assume that the initial capacitor voltage is zero.

Solution:
Since $v = \frac{1}{C}\int_0^t i d\tau + v(0)$ and $v(0) = 0$,

$$v = \frac{1}{2\times10^{-6}}\int_0^t 6e^{-3000\tau}\,d\tau\cdot10^{-3}$$
$$= \frac{3\times10^3}{-3000}e^{-3000\tau}\Big|_0^t = (1 - e^{-3000t})\text{ V}$$

Example 1.21
Find the equivalent capacitance seen between terminals *a* and *b* of the circuit shown.



Solution:
The 20-μF and 5-μF capacitors are in series

$$\frac{20 \times 5}{20+5} = 4\ \mu\text{F}$$

This 4-μF capacitor is in parallel with the 6-μF and 20-μF capacitors;

$$4 + 6 + 20 = 30\ \mu\text{F}$$

This 30-μF capacitor is in series with the 60-μF capacitor. Hence, the equivalent capacitance for the entire circuit is

$$C_{eq} = \frac{30 \times 60}{30+60} = 20\ \mu\text{F}$$

Example 1.22

The current through a 0.1-H inductor is $i(t) = 10te^{-5t}$ A. Find the voltage across the inductor and the energy stored in it.

Solution:

Since $v = L\, di/dt$ and $L = 0.1$ H,

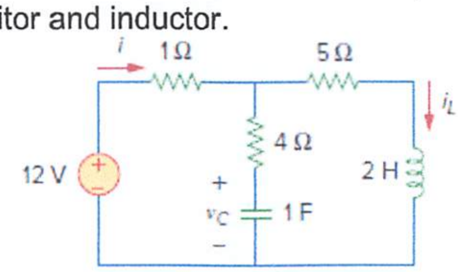
$$v = 0.1\frac{d}{dt}(10te^{-5t}) = e^{-5t} + t(-5)e^{-5t} = e^{-5t}(1 - 5t)\ \text{V}$$

The energy stored is

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)100t^2e^{-10t} = 5t^2e^{-10t}\ \text{J}$$

Example 1.23

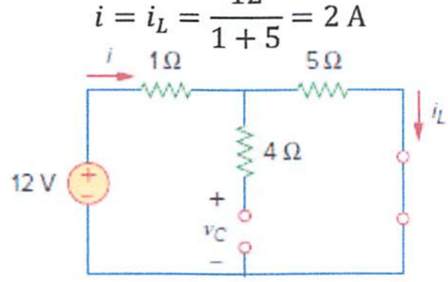
Consider the circuit shown below. Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.



Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in the figure below.

$$i = i_L = \frac{12}{1+5} = 2\ \text{A}$$



The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10\text{ V}$$

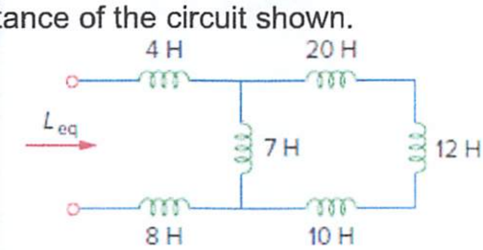
(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50\text{ J}$$

and the inductor

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4\text{ J}$$

Example 1.24
Find the equivalent inductance of the circuit shown.



Solution:
The 10-H, 12-H, and 20-H inductors are in series; thus, combining them gives a 42-H inductance. This 42-H inductor is in parallel with the 7-H inductor so that they are combined, to give

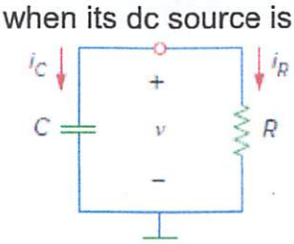
$$\frac{7 \times 42}{7 + 42} = 6\text{ H}$$

This 6-H inductor is in series with the 4-H and 8-H inductors. Hence,

$$L_{eq} = 4 + 6 + 8 = 18\text{ H}$$

1.6 FIRST-ORDER CIRCUITS
First-order circuit is characterized by a first-order differential equation.

The Source-Free RC Circuit:
A **source-free *RC*** circuit occurs when its dc source is suddenly disconnected.



The **natural response** of a circuit refers to the behavior (in terms of voltages and currents) of the circuit itself, with no external sources of excitation.
The **voltage response** or **natural response** of the ***RC*** circuit is an exponential decay of the initial voltage:

$$v(t) = V_0 e^{-t/RC}$$

The **time constant** of a circuit is the time required for the response to decay to a factor of 1/e or 36.8 percent of its initial value.

$$\tau = RC$$

In terms of the time constant, the voltage response can be written as

$$v(t) = V_0 e^{-t/\tau}$$

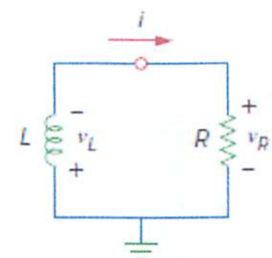
With the voltage *v*(*t*), we can find the current $i_R(t)$,

$$i_R(t) = \frac{v(t)}{R} = \frac{V_0}{R} e^{-t/\tau}$$

The key to working with a source-free RC circuit is finding:

1. The initial voltage *v*(0) = V_0 across the capacitor.
2. The time constant τ.

The Source-Free RL Circuit:



The **natural response** of the ***RL*** circuit is an exponential decay of the initial current.

$$i(t) = I_0 e^{-Rt/L}$$

The time constant for the *RL* circuit is

$$\tau = \frac{L}{R}$$

The natural response of an RL circuit can be written as

$$i(t) = I_0 e^{-t/\tau}$$

With the current, we can find the voltage across the resistor as

$$v_R(t) = iR = I_0 R e^{-t/\tau}$$

The Key to Working with a Source-Free RL Circuit Is to Find:

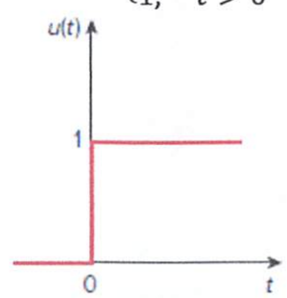
1. The initial current *i*(0) = I_0 through the inductor.
2. The time constant τ of the circuit.

Singularity functions are functions that either are discontinuous or have discontinuous derivatives.

The three most widely used singularity functions in circuit analysis are the *unit step*, the *unit impulse*, and the *unit ramp* functions.

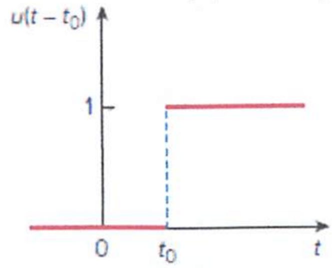
The **unit step function** u(t) is 0 for negative values of t and 1 for positive values of t.

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$



If the abrupt change occurs at $t = t_0$ (where $t_0 > 0$) instead of $t = 0$, the unit step function becomes

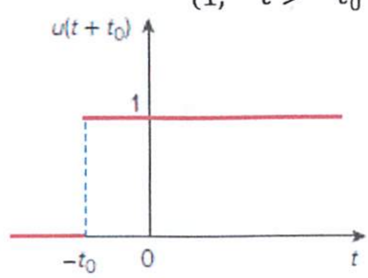
$$u(t - t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t > t_0 \end{cases}$$



which is the same as saying that $u(t)$ is delayed by t_0 seconds.

If the change is at $t = -t_0$, the unit step function becomes

$$u(t + t_0) = \begin{cases} 0, & t < -t_0 \\ 1, & t > -t_0 \end{cases}$$



meaning that $u(t)$ is advanced by t_0 seconds.

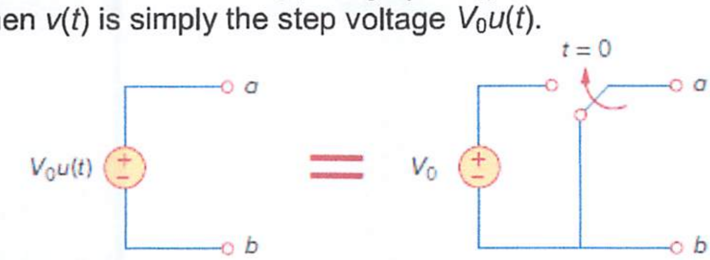
We use the step function to represent an abrupt change in voltage or current. For example, the voltage

$$v(t) = \begin{cases} 0, & t < t_0 \\ V_0, & t > t_0 \end{cases}$$

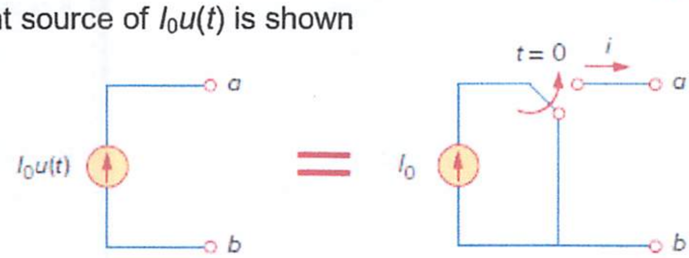
may be expressed in terms of the unit step function as

$$v(t) = V_0 u(t - t_0)$$

If we let $t_0 = 0$, then $v(t)$ is simply the step voltage $V_0u(t)$.



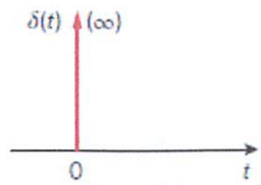
Similarly, a current source of $I_0u(t)$ is shown



The derivative of the unit step function *u*(*t*) is the ***unit impulse function δ(t)***.

$$\delta(t) = \frac{d}{dt}u(t) = \begin{cases} 0, & t < 0 \\ \text{Undefined}, & t = 0 \\ 0, & t > 0 \end{cases}$$

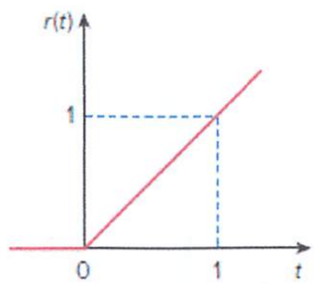
The **unit impulse function** δ(t) is zero everywhere except at t = 0, where it is undefined.



Integrating the unit step function *u*(*t*) results in the ***unit ramp function r(t).***

$$r(t) = \begin{cases} 0, & t \le 0 \\ t, & t \ge 0 \end{cases}$$

The **unit ramp function** is zero for negative values of t and has a unit slope for positive values of t.



For the delayed unit ramp function,

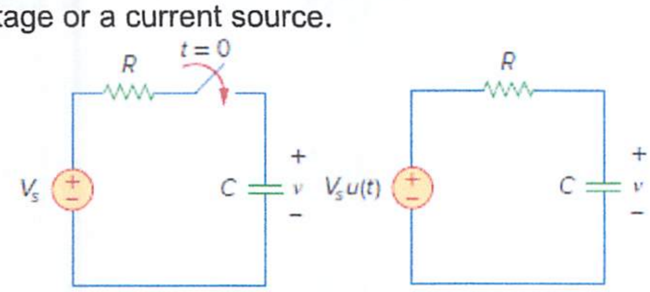
$$r(t - t_0) = \begin{cases} 0, & t \le t_0 \\ t - t_0, & t \ge t_0 \end{cases}$$

For the advanced unit ramp function,

$$r(t + t_0) = \begin{cases} 0, & t \le -t_0 \\ t + t_0, & t \ge -t_0 \end{cases}$$

Step Response of an RC Circuit:

The **step response** of a circuit is its behavior when the excitation is the step function, which may be a voltage or a current source.



The step response is the response of the circuit due to a sudden application of a dc voltage or current source.

The ***complete response*** (or total response) of the *RC* circuit to a sudden application of a dc voltage source, assuming the capacitor is initially charged.

$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau}, & t > 0 \end{cases}$$

$$\underset{\text{stored energy}}{\text{Complete response} = \text{natural response}} + \underset{\text{independent source}}{\text{forced response}}$$

or

$$v = v_n + v_f$$

where

$$v_n = V_0 e^{-t/\tau}$$

and

$$v_f = V_s(1 - e^{-t/\tau})$$

Another way of looking at the complete response is to break into two components—one temporary and the other permanent, that is

$$\underset{\text{temporary part}}{\text{Complete response} = \text{transient response}} + \underset{\text{permanent part}}{\text{steady-state response}}$$

or

$$v = v_t + v_{ss}$$

where

$$v_t = (V_0 - V_s)e^{-t/\tau}$$

and

$$v_{ss} = V_s$$

The **transient response** is the circuit's temporary response that will die out with time.
The **steady-state response** is the behavior of the circuit a long time after an external excitation is applied.

The complete response can now be written as

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

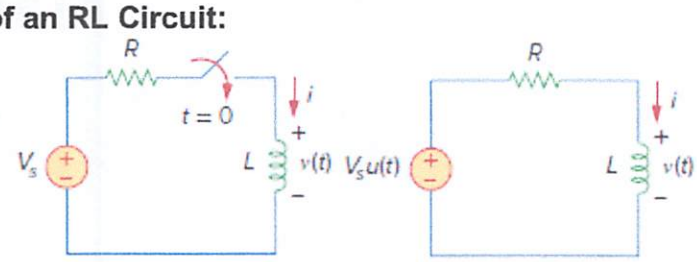
If the switch changes position at time $t = t_0$ instead of at $t = 0$ the equation becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau}$$

To find the step response of an *RC* circuit requires three things:

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

Step Response of an RL Circuit:



The complete response of the *RL* circuit is

$$i(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

or

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

If the switching takes place at time $t = t_0$ instead of $t = 0$ the equation becomes

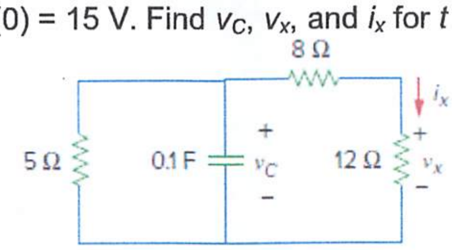
$$i(t) = i(\infty) + [i(t_0) - i(\infty)]e^{-(t-t_0)/\tau}$$

To find the step response of an *RL* circuit requires three things:

1. The initial inductor current $i(0)$ at $t = 0$.
2. The final inductor current $i(\infty)$.
3. The time constant τ.

Example 1.25

In the figure shown, let $v_C(0) = 15$ V. Find v_C, v_x, and i_x for $t > 0$.



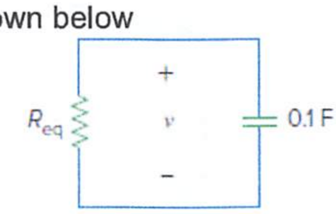
Solution:

We find the equivalent resistance or the Thevenin resistance at the capacitor terminals. Our objective is always to first obtain capacitor voltage v_C. From this, we can determine v_x and i_x.

The 8-Ω and 12-Ω resistors in series can be combined to give a 20-Ω resistor. This 20-Ω resistor in parallel with the 5-Ω resistor can be combined so that the equivalent resistance is

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

The equivalent circuit is as shown below



The time constant is

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{S}$$

Thus,

$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}, \qquad v_C = v = 15e^{-2.5t}\ \text{V}$$

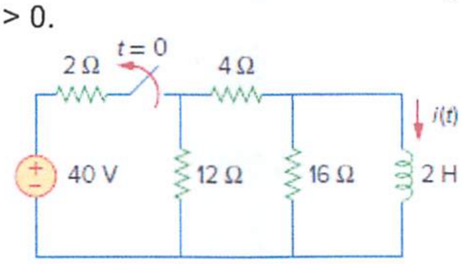
From the original figure, we can use voltage division to get v_x; so

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

Finally,

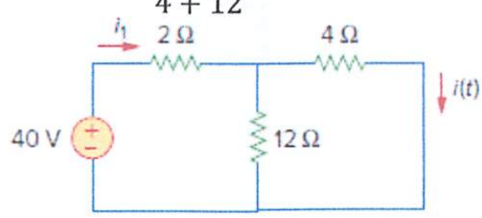
$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

Example 1.26
The switch in the circuit shown has been closed for a long time. At *t* = 0, the switch is opened. Calculate *i*(*t*) for *t* > 0.



Solution:
When *t* < 0, the switch is closed, and the inductor acts as a short circuit to dc. The 16-Ω resistor is short-circuited; the resulting circuit is shown in Fig. (a). To get i_1 in Fig. (a), we combine the 4-Ω and 12-Ω resistors in parallel to get

$$\frac{4 \times 12}{4 + 12} = 3\ \Omega$$



(a)

Hence,

$$i_1 = \frac{40}{20 + 3} = 8\ \text{A}$$

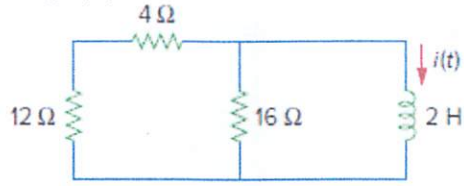
We obtain *i*(*t*) from i_1 in Fig. (a) using current division

$$i(t) = \frac{12}{12 + 4} i_1 = 6\ \text{A}, \qquad t < 0$$

Since the current through an inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6\ \text{A}$$

When *t* > 0, the switch is open and the voltage source is disconnected. We now have the source-free *RL* circuit in Fig. (b).



(b)

Combining the resistors

$$R_{eq} = (12 + 4) \parallel 16 = 8\,\Omega$$

The time constant is

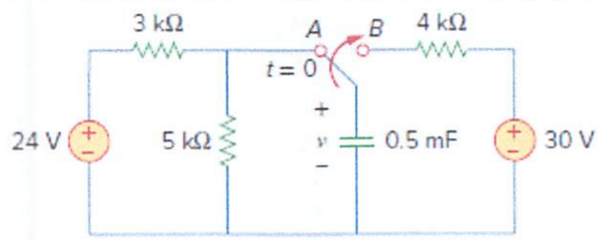
$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\text{S}$$

Thus,

$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\ \text{A}$$

Example 1.27

The switch in the figure has been in position *A* for a long time. At $t = 0$, the switch moves to *B*. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ and 4 s.



Solution:

For $t < 0$, the switch is at position *A*. The capacitor acts like an open circuit to dc, but *v* is the same as the voltage across the 5-kΩ resistor.

Hence, the voltage across the capacitor just before $t = 0$ is

$$v(0^-) = \frac{5}{5+3}(24) = 15\ \text{V}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15\ \text{V}$$

For $t > 0$, the switch is in position *B*. The Thevenin resistance connected to the capacitor is $R_{Th} = 4\ \text{k}\Omega$,

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2\ \text{s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30$ V. Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t})\text{V}$$

At t=1,

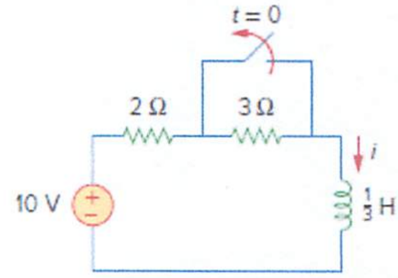
$$v(1) = 30 - 15e^{-0.5} = 20.9\ \text{V}$$

At t=4,

$$v(4) = 30 - 15e^{-2} = 27.97\ \text{V}$$

Example 1.28

Find $i(t)$ in the circuit shown for $t > 0$. Assume that the switch has been closed for a long time.



Solution:

When $t < 0$, the 3-Ω resistor is short-circuited, and the inductor acts like a short circuit. The current through the inductor at $t = 0^-$

$$i(0^-) = \frac{10}{2} = 5\text{ A}$$

Because the inductor current cannot change instantaneously,

$$i(0) = i(0^+) = i(0^-) = 5\text{ A}$$

When $t > 0$, the switch is open. The 2- and 3-Ω resistors are in series

$$i(\infty) = \frac{10}{2+3} = 2\text{ A}$$

The Thevenin resistance across the inductor terminals is

$$R_{\text{Th}} = 2 + 3 = 5\ \Omega$$

For the time constant,

$$\tau = \frac{L}{R_{\text{Th}}} = \frac{1/3}{5} = \frac{1}{15}\text{s}$$

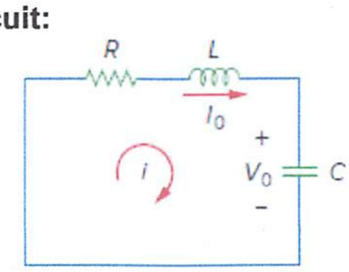
Thus,

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$
$$= 2 + (5-2)e^{-15t} = 2 + 3e^{-15t}\text{ A}, \qquad t > 0$$

1.7 SECOND-ORDER CIRCUITS

A **second-order circuit** is characterized by a second-order differential equation. It consists of resistors and the equivalent of two energy storage elements.
Typical examples of second-order circuits are *RLC* circuits.

Source-Free Series RLC Circuit:



Characteristic equation:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

Roots of the characteristic equation:

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \qquad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{R}{2L}, \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

The natural response of the series *RLC* circuit is

$$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$$

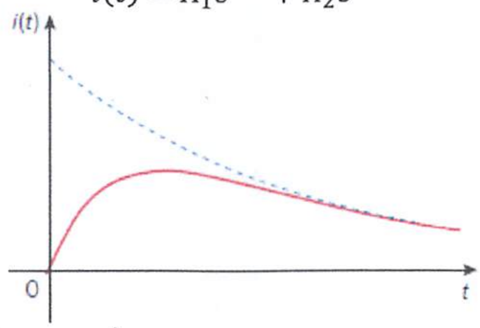
Three types of solutions:

1. If $\alpha > \omega_0$, we have the *overdamped* case.
2. If $\alpha = \omega_0$, we have the *critically damped* case.

3. If $\alpha < \omega_0$, we have the *underdamped* case.

Overdamped Case ($\alpha > \omega_0$)
When $\alpha > \omega_0$, $C > 4L/R^2$. Both roots s_1 and s_2 are negative and real. The response is

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

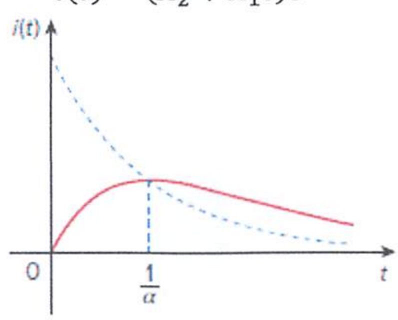


Critically Damped Case ($\alpha = \omega_0$)
When $\alpha = \omega_0$, $C = 4L/R^2$ and

$$s_1 = s_2 = -\alpha = -\frac{R}{2L}$$

The response is

$$i(t) = (A_2 + A_1 t)e^{-\alpha t}$$



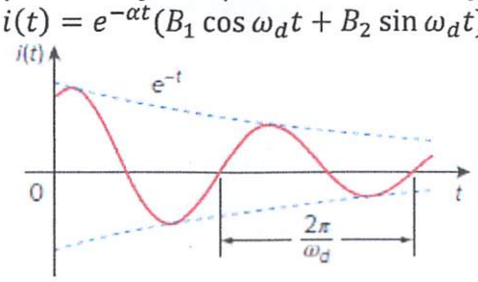
Underdamped Case ($\alpha < \omega_0$)
For $\alpha < \omega_0$, $C < 4L/R^2$. The roots may be written as

$$s_1 = -\alpha + \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha + j\omega_d$$

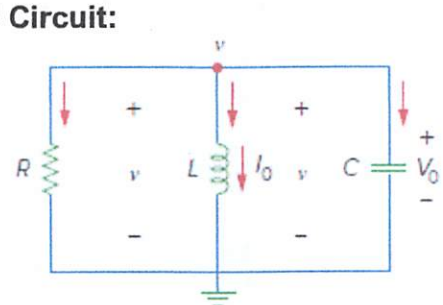
$$s_2 = -\alpha - \sqrt{-(\omega_0^2 - \alpha^2)} = -\alpha - j\omega_d$$

where $j = \sqrt{-1}$ and $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$, which is called the *damped frequency*.
The natural response is exponentially damped and oscillatory in nature.

$$i(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$$



Source-Free Parallel RLC Circuit:



Characteristic equation:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

Roots of the characteristic equation:

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \text{ or } s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Overdamped Case ($\alpha > \omega_0$)
When $\alpha > \omega_0$, $L > 4R^2C$. The roots of the characteristic equation are real and negative. The response is

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Critically Damped Case ($\alpha = \omega_0$)
For $\alpha = \omega_0$, $L = 4R^2C$. The roots are real and equal so that the response is

$$v(t) = (A_1 + A_2 t)e^{-\alpha t}$$

Underdamped Case ($\alpha < \omega_0$)
When $\alpha < \omega_0$, $L < 4R^2C$. In this case the roots are complex and may be expressed as

$$s_{1,2} = -\alpha \pm j\omega_d$$

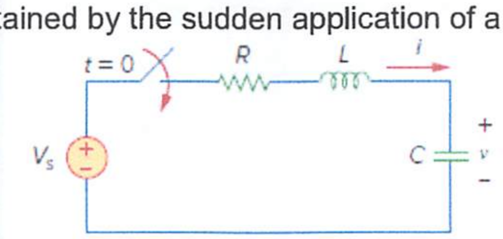
where

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

The response is

$$v(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

Step Response of a Series RLC Circuit:
The step response is obtained by the sudden application of a dc source.



Characteristic equation:

$$\frac{d^2v}{dt^2}+\frac{R}{L}\frac{dv}{dt}+\frac{v}{LC}=\frac{V_s}{LC}$$

The complete solution of the equation consists of the transient response and steady-state response:

$$v(t) = v_t(t) + v_{ss}(t)$$

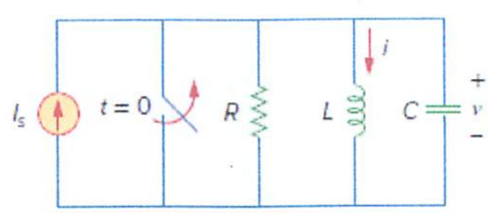
The complete solutions for the overdamped, underdamped, and critically damped cases are:

$$v(t) = V_s + A_1e^{s_1t} + A_2e^{s_2t} \quad \text{(Overdamped)}$$

$$v(t) = V_s + (A_1 + A_2t)e^{-\alpha t} \quad \text{(Critically damped)}$$

$$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \quad \text{(Underdamped)}$$

Step Response of a Parallel RLC Circuit:



Characteristic equation:

$$\frac{d^2i}{dt^2}+\frac{1}{RC}\frac{di}{dt}+\frac{i}{LC}=\frac{I_s}{LC}$$

The complete solution of the equation:

$$i(t) = i_t(t) + i_{ss}(t)$$

The complete solutions for the overdamped, underdamped, and critically damped cases are:

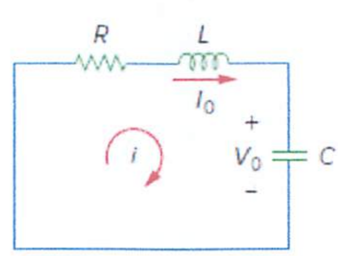
$$i(t) = I_s + A_1e^{s_1t} + A_2e^{s_2t} \quad \text{(Overdamped)}$$

$$i(t) = I_s + (A_1 + A_2t)e^{-\alpha t} \quad \text{(Critically damped)}$$

$$i(t) = I_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t} \quad \text{(Underdamped)}$$

Example 1.29

In the figure shown, R = 40 Ω, L = 4 H, and C = 1/4 F. Calculate the characteristic roots of the circuit. Is the natural response overdamped, underdamped, or critically damped?



Solution:

$$\alpha = \frac{R}{2L} = \frac{40}{2(4)} = 5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times \frac{1}{4}}} = 1$$

The roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{25 - 1}$$

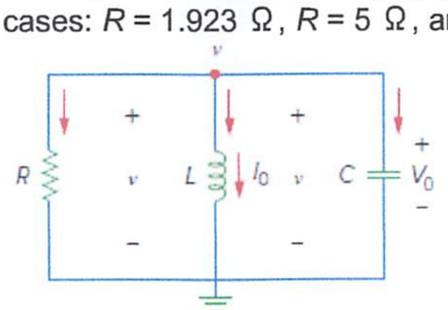
or

$$s_1 = -0.101, \quad s_2 = -9.899$$

Since $\alpha > \omega_0$, we conclude that the response is overdamped. This is also evident from the fact that the roots are real and negative.

Example 1.30

In the parallel circuit shown, find $v(t)$ for $t > 0$, assuming $v(0) = 5$ V, $i(0) = 0$, $L = 1$ H, and $C = 10$ mF. Consider these cases: $R = 1.923\ \Omega$, $R = 5\ \Omega$, and $R = 6.25\ \Omega$.



Solution:

CASE 1 If $R = 1.923\ \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 1.923 \times 10 \times 10^{-3}} = 26$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 10 \times 10^{-3}}} = 10$$

Since $\alpha > \omega_0$ in this case, the response is overdamped.

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -2, -50$$

the corresponding response is

$$v(t) = A_1 e^{-2t} + A_2 e^{-50t} \qquad (1)$$

Applying the initial conditions to get A_1 and A_2.

$$v(0) = 5 = A_1 + A_2 \qquad (2)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5 + 0}{1.923 \times 10 \times 10^{-3}} = -260$$

Differentiating Eq. (1),

$$\frac{dv}{dt} = -2A_1 e^{-2t} - 50A_2 e^{-50t}$$

At $t = 0$,

$$-260 = -2A_1 - 50A_2 \quad (3)$$

From Eqs. (2) and (3), we obtain $A_1 = -0.2083$ and $A_2 = 5.208$. Substituting A_1 and A_2 in Eq. (1) yields

$$v(t) = -0.2083e^{-2t} + 5.208e^{-50t} \quad (4)$$

CASE 2 When $R = 5\ \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 5 \times 10 \times 10^{-3}} = 10$$

while $\omega_0 = 10$ remains the same. Since $\alpha = \omega_0 = 10$, the response is critically damped. Hence, $s_1 = s_2 = -10$, and

$$v(t) = (A_1 + A_2 t)e^{-10t} \quad (5)$$

$$v(0) = 5 = A_1 \quad (6)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5+0}{5 \times 10 \times 10^{-3}} = -100$$

Differentiating Eq. (5),

$$\frac{dv}{dt} = (-10A_1 - 10A_2 t + A_2)e^{-10t}$$

At $t = 0$,

$$-100 = -10A_1 + A_2 \quad (7)$$

From Eqs. (6) and (7), $A_1 = 5$ and $A_2 = -50$. Thus,

$$v(t) = (5 - 50t)e^{-10t}\ \text{V} \quad (8)$$

CASE 3 When $R = 6.25\ \Omega$,

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 6.25 \times 10 \times 10^{-3}} = 8$$

while $\omega_0 = 10$ remains the same. As $\alpha < \omega_0$ in this case, the response is underdamped. The roots of the characteristic equation are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -8 \pm j6$$

Hence,

$$v(t) = (A_1 \cos 6t + A_2 \sin 6t)e^{-8t} \quad (9)$$

$$v(0) = 5 = A_1 \quad (10)$$

$$\frac{dv(0)}{dt} = -\frac{v(0) + Ri(0)}{RC} = -\frac{5+0}{6.25 \times 10 \times 10^{-3}} = -80$$

Differentiating Eq. (9),

$$\frac{dv}{dt} = (-8A_1 \cos 6t - 8A_2 \sin 6t - 6A_1 \sin 6t + 6A_2 \cos 6t)e^{-8t}$$

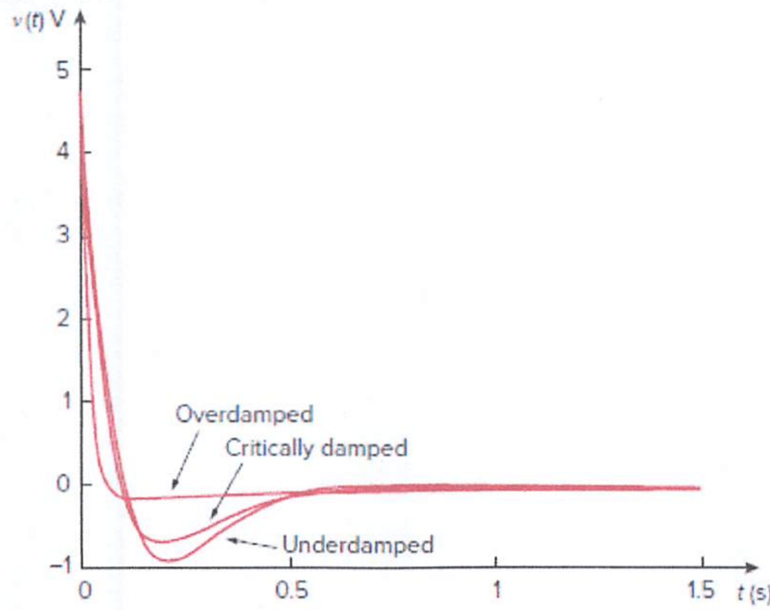
At $t = 0$,

$$-80 = -8A_1 + 6A_2 \quad (11)$$

From Eqs. (10) and (11), $A_1 = 5$ and $A_2 = -6.667$. Thus,

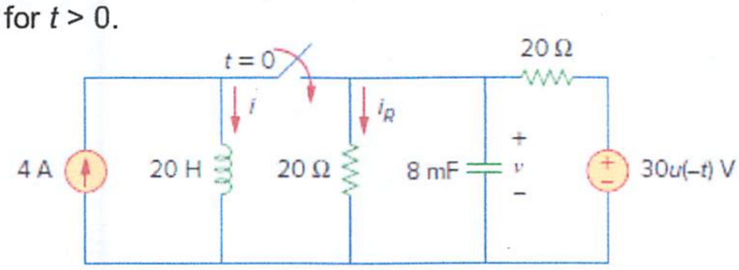
$$v(t) = (5\cos 6t - 6.667\sin 6t)e^{-8t} \tag{12}$$

Figure below plots the three cases.



Example 1.31

Find $i(t)$ and $i_R(t)$ for $t > 0$.



Solution:

For $t < 0$, the switch is open, and the circuit is partitioned into two independent subcircuits. The 4-A current flows through the inductor, so that

$$i(0) = 4\text{ A}$$

Since $30u(-t) = 30$ when $t < 0$ and 0 when $t > 0$, the voltage source is operative for $t < 0$. The capacitor acts like an open circuit and the voltage across it is the same as the voltage across the 20-Ω resistor connected in parallel with it. By voltage division, the initial capacitor voltage is

$$v(0) = \frac{20}{20+20}(30) = 15\text{ V}$$

For $t > 0$, the switch is closed, and we have a parallel *RLC* circuit with a current source. The voltage source is zero which means it acts like a short-circuit. The two 20-Ω resistors are now in parallel. They are combined to give $R = 20 \parallel 20 = 10\ \Omega$. The characteristic roots are determined as follows:

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 10\times 8\times 10^{-3}} = 6.25$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 8 \times 10^{-3}}} = 2.5$$

$$s_{1,2} = \alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm \sqrt{39.0625 - 6.25} = -6.25 \pm 5.7282$$

or

$$s_1 = -11.978, \quad s_2 = -0.5218$$

Since $\alpha > \omega_0$, we have the overdamped case. Hence,

$$i(t) = I_s + A_1 e^{-11.978t} + A_2 e^{-0.5218t} \tag{1}$$

where I_s = 4 is the final value of $i(t)$. We now use the initial conditions to determine A_1 and A_2. At t = 0,

$$i(0) = 4 = 4 + A_1 + A_2 \quad \Rightarrow \quad A_2 = -A_1 \tag{2}$$

Taking the derivative of $i(t)$ in Eq. (1),

$$\frac{di}{dt} = -11.978 A_1 e^{-11.978t} - 0.5218 A_2 e^{-0.5218t}$$

so that at t = 0,

$$\frac{di(0)}{dt} = -11.978 A_1 - 0.5218 A_2 \tag{3}$$

But

$$L\frac{di(0)}{dt} = v(0) = 15 \quad \Rightarrow \quad \frac{di(0)}{dt} = \frac{15}{L} = \frac{15}{20} = 0.75$$

Substituting this into Eq. (3) and incorporating Eq. (2), we get

$$0.75 = (11.978 - 0.5218)A_2 \Rightarrow A_2 = 0.0655$$

Thus, A_1 = −0.0655 and A_2 = 0.0655. Inserting A_1 and A_2 in Eq. (1) gives the complete solution as

$$i(t) = 4 + 0.0655(e^{-0.5218t} - e^{-11.978t})\text{A}$$

From $i(t)$, we obtain $v(t) = L\, di/dt$ and

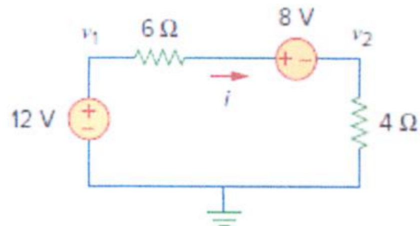
$$i_R(t) = \frac{v(t)}{20} = \frac{L}{20}\frac{di}{dt} = 0.785e^{-11.978t} - 0.0342e^{-0.5218t}\text{A}$$

Self-Evaluation:

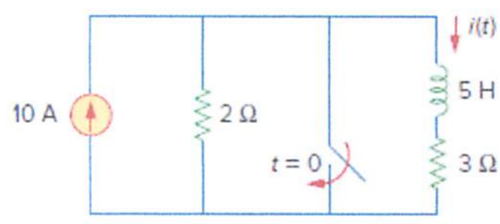
1. Which of these is not an electrical quantity?
   a. charge
   b. time
   c. voltage
   d. current
   e. power
2. Voltage is measured in:
   a. watts
   b. amperes
   c. volts
   d. joules per second
3. A network has 12 branches and 8 independent loops. How many nodes are there in the network?
   a. 17

b. 19
c. 4
d. 5

4. The maximum current that a 2W, 80 kΩ resistor can safely conduct is:
   a. 40 kA
   b. 160 kA
   c. 25 mA
   d. 5 μA
5. In the circuit shown, the voltage v_2 is:

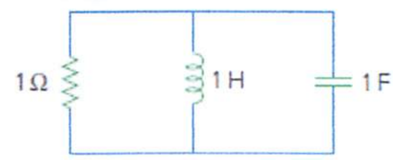


   a. 8 V
   b. 1.6 V
   c. −1.6 V
   d. −8 V
6. The Norton resistance R_N is exactly equal to the Thevenin resistance R_{Th}.
   a. True
   b. False
7. The superposition principle applies to power calculation.
   a. True
   b. False
8. When the total charge in a capacitor is doubled, the energy stored:
   a. remains the same
   b. is doubled
   c. is halved
   d. is quadrupled
9. Inductors in parallel can be combined just like resistors in parallel.
   a. True
   b. False
10. A capacitor in an RC circuit with R = 2 Ω and C = 4 F is being charged. The time required for the capacitor voltage to reach 63.2 percent of its steady-state value is:
   a. 8 s
   b. 4 s
   c. 2 s
   d. 16 s
11. For the circuit shown, the inductor current just before t = 0 is:



   a. 2 A
   b. 6 A
   c. 4 A
   d. 8 A
12. Consider the parallel RLC circuit shown. What type of response will it produce?
   a. underdamped

b. overdamped
c. critically damped
d. none of the above



13. A parallel *RLC* circuit has L = 2 H and C = 0.25 F. The value of R that will produce a unity neper frequency is:
    a. 0.5 Ω
    b. 4 Ω
    c. 2 Ω
    d. 1 Ω
14. When a step input is applied to a second-order circuit, the final values of the circuit variables are found by:
    a. Replacing capacitors with closed circuits and inductors with open circuits.
    b. Replacing capacitors with open circuits and inductors with closed circuits.
    c. Doing neither of the above.

Review of Concepts:

1. An electric circuit consists of electrical elements connected together.
2. Current is the rate of charge flow past a given point in a given direction.
3. Voltage is the energy required to move 1 C of charge from a reference point (−) to another point (+).
4. Power is the energy supplied or absorbed per unit time. It is also the product of voltage and current.
5. According to the passive sign convention, power assumes a positive sign when the current enters the positive polarity of the voltage across an element.
6. An ideal voltage source produces a specific potential difference across its terminals regardless of what is connected to it. An ideal current source produces a specific current through its terminals regardless of what is connected to it.
7. Voltage and current sources can be dependent or independent. A dependent source is one whose value depends on some other circuit variable.
8. A resistor is a passive element in which the voltage v across it is directly proportional to the current i through it. That is, a resistor is a device that obeys Ohm's law.
9. A short circuit is a resistor (a perfectly, conducting wire) with zero resistance (R = 0). An open circuit is a resistor with infinite resistance (R = ∞).
10. The conductance G of a resistor is the reciprocal of its resistance. A branch is a single two-terminal element in an electric circuit. A node is the point of connection between two or more branches. A loop is a closed path in a circuit.
11. Kirchhoff's current law (KCL) states that the currents at any node algebraically sum to zero. In other words, the sum of the currents entering a node equals the sum of currents leaving the node.
12. Kirchhoff's voltage law (KVL) states that the voltages around a closed path algebraically sum to zero. In other words, the sum of voltage rises equals the sum of voltage drops.
13. Two elements are in series when they are connected sequentially, end to end.

When elements are in series, the same current flows through them ($i_1 = i_2$). They are in parallel if they are connected to the same two nodes. Elements in parallel always have the same voltage across them ($v_1 = v_2$).

14. Nodal analysis is the application of Kirchhoff's current law at the nonreference nodes. (It is applicable to both planar and nonplanar circuits.) We express the result in terms of the node voltages. Solving the simultaneous equations yields the node voltages.
15. A supernode consists of two nonreference nodes connected by a (dependent or independent) voltage source.
16. Mesh analysis is the application of Kirchhoff's voltage law around meshes in a planar circuit. We express the result in terms of mesh currents. Solving the simultaneous equations yields the mesh currents.
17. A supermesh consists of two meshes that have a (dependent or independent) current source in common.
18. A linear network consists of linear elements, linear dependent sources, and linear independent sources.
19. Network theorems are used to reduce a complex circuit to a simpler one, thereby making circuit analysis much simpler.
20. The superposition principle states that for a circuit having multiple independent sources, the voltage across (or current through) an element is equal to the algebraic sum of all the individual voltages (or currents) due to each independent source acting one at a time.
21. Source transformation is a procedure for transforming a voltage source in series with a resistor to a current source in parallel with a resistor, or vice versa.
22. Thevenin's and Norton's theorems allow us to isolate a portion of a network while the remaining portion of the network is replaced by an equivalent network. The Thevenin equivalent consists of a voltage source V_{Th} in series with a resistor R_{Th}, while the Norton equivalent consists of a current source I_N in parallel with a resistor R_N.
23. For a given Thevenin equivalent circuit, maximum power transfer occurs when $R_L = R_{Th}$; that is, when the load resistance is equal to the Thevenin resistance.
24. The current through a capacitor is directly proportional to the time rate of change of the voltage across it. The current through a capacitor is zero unless the voltage is changing. Thus, a capacitor acts like an open circuit to a dc source.
25. The voltage across a capacitor is directly proportional to the time integral of the current through it. The voltage across a capacitor cannot change instantly.
26. Capacitors in series and in parallel are combined in the same way as conductances.
27. The voltage across an inductor is directly proportional to the time rate of change of the current through it. The voltage across the inductor is zero unless the current is changing. Thus, an inductor acts like a short circuit to a dc source.
28. The current through an inductor is directly proportional to the time integral of the voltage across it. The current through an inductor cannot change instantly.
29. Inductors in series and in parallel are combined in the same way resistors in series and in parallel are combined.

30. At any given time t, the energy stored in a capacitor is $\frac{1}{2}Cv^2$, while the energy stored in an inductor is $\frac{1}{2}Li^2$.
31. The natural response is obtained when no independent source is present.
32. The time constant τ is the time required for a response to decay to 1/e of its initial value. For RC circuits, τ = RC and for RL circuits, τ = L/R.
33. The singularity functions include the unit step, the unit ramp function, and the unit impulse functions.
34. The steady-state response is the behavior of the circuit after an independent source has been applied for a long time. The transient response is the component of the complete response that dies out with time.
35. The total or complete response consists of the steady-state response and the transient response.
36. The step response is the response of the circuit to a sudden application of a dc current or voltage.
37. The determination of the initial values x(0) and dx(0)/dt and final value x(∞) is crucial to analyzing second-order circuits.
38. The RLC circuit is second-order because it is described by a second-order differential equation.
39. If there are no independent sources in the circuit after switching (or sudden change), we regard the circuit as source-free. The complete solution is the natural response.
40. The natural response of an RLC circuit is overdamped, underdamped, or critically damped, depending on the roots of the characteristic equation.
41. If independent sources are present in the circuit after switching, the complete response is the sum of the transient response and the steady-state response.

References:

- Charles Alexander, Matthew Sadiku-*Fundamentals of Electric Circuits* (McGraw-Hill Education; 6th ed. 2016)
- Mahmood Nahvi, PhD. & Joseph A. Edminister- *Schaum's Outlines of Electric Circuits* (McGraw-Hill Education; 7th ed. 2017)