SURIGAO STATE COLLEGE OF TECHNOLOGY

SSCT

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I.14. The faculty are encouraged to produce their own instructional materials such as modules, software, visual aids, manuals and textbooks.

Republic of the Philippines
SURIGAO STATE COLLEGE OF TECHNOLOGY
Narciso Street, Surigao City
"For Nation's Greater Heights"
CERTIFICATE NUMBER: AJA19-0225

OFFICE OF THE PRESIDENT

OFFICE ORDER # 182

SSCT SURIGAO CITY
RELEASED
DATE: 11-22-19
NAME:

REFERENCE NO.	:	**SSCT - OP – 11 –05 Series 2019**
DATE	:	**November 21, 2019**
TO	:	**ALL FACULTY MEMBERS**
FROM	:	**THE COLLEGE PRESIDENT**
SUBJECT	:	**SUBMISSION OF THE INSTRUCTIONAL MATERIALS**

You are hereby advised to submit your **INSTRUCTIONAL MATERIALS** for review of external reviewers and evaluators to the Office of the Vice President for Academic Affairs **not later than November 29, 2019.**

For your strict compliance.

GREGORIO Z. GAMBOA, JR., EdD
SUC President III

cc.

VPs
College Deans
Campus Director

Module no. 1
Sinusoidal Steady-State Analysis

Topic: 1.1 Nodal and Mesh Analysis
- 1.1.1 Nodal Analysis
- 1.1.2 Mesh Analysis

1.2 Superposition Theorem
1.3 Source Transformation
1.4 Thevenin's and Norton's Theorems

Time Frame: 4 hrs.

Introduction:
In this module, we want to see how nodal analysis, mesh analysis, Thevenin's theorem, Norton's theorem, superposition, and source transformations are applied in analyzing ac circuits. Since these techniques were already introduced for dc circuits, our major effort here will be to illustrate with examples.

Analyzing ac circuits usually requires three steps:
1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.

Step 1 is not necessary if the problem is specified in the frequency domain. In step 2, the analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.

Objectives:
At the end of this topic, the student shall be able to
1. Analyze electrical circuits in the frequency domain using nodal and mesh analysis.
2. Apply the superposition principle to frequency domain electrical circuits.
3. Apply source transformation in frequency domain circuits.
4. Understand how Thevenin and Norton equivalent circuits can be used in the frequency domain.

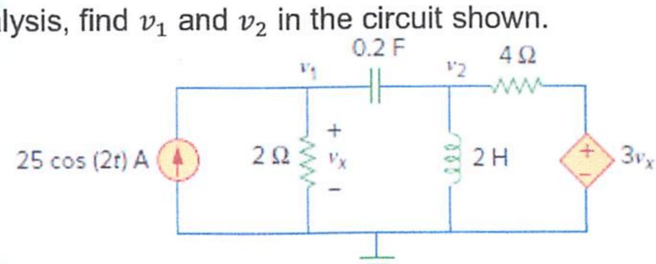
Pre – Test

Module 1 – Sinusoidal Steady-State Analysis

Name: **Subject:**
Course/Section: **Date:**

Direction: Read the problems carefully. Write your solutions in a separate sheet of paper.

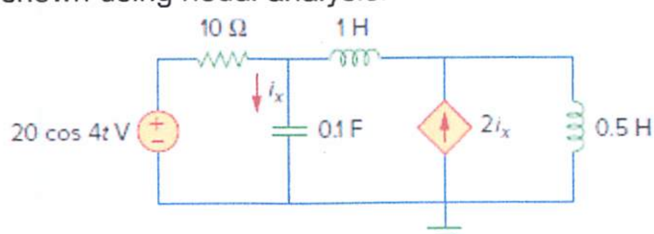
1. What is the basis for nodal and mesh analysis?
2. Can we use all the circuit analysis techniques to ac circuits the same way they are used in dc circuits?
3. Using nodal analysis, find v_1 and v_2 in the circuit shown.



4. Is frequency domain analysis of an ac circuit via time domain much complicated than the analysis of the circuit via phasors?

Learning Activities:
1.1 NODAL AND MESH ANALYSIS
1.1.1 Nodal Analysis
The basis of nodal analysis is Kirchhoff's current law. Since KCL is valid for phasors, we can analyze ac circuits by nodal analysis.

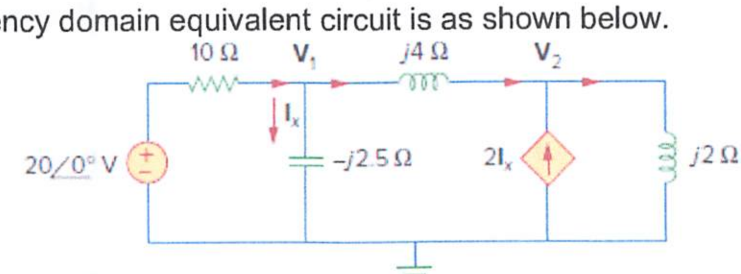
Example 1.1
Find i_x in the circuit shown using nodal analysis.



Solution:
We first convert the circuit to the frequency domain:

$$\begin{array}{lcll} 20\cos 4t & \Rightarrow & 20\angle 0^\circ, & \Omega = 4 \text{ rad/s} \\ 1\text{ H} & \Rightarrow & j\Omega L = j4 & \\ 0.5\text{ H} & \Rightarrow & j\Omega L = j2 & \\ 0.1\text{ F} & \Rightarrow & \dfrac{1}{j\Omega C} = -j2.5 & \end{array}$$

Thus, the frequency domain equivalent circuit is as shown below.



Applying KCL at node 1,

$$\frac{20 - V_1}{10} = \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4}$$

Or

$$(1 + j1.5)V_1 + j2.5V_2 = 20 \tag{1}$$

At node 2,

$$2I_x + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$$

But $I_x = V_1 / -j2.5$. Substituting this gives

$$\frac{2V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = \frac{V_2}{j2}$$

By simplifying, we get

$$11V_1 + 15V_2 = 0 \tag{2}$$

Equations (1) and (2) can be put in matrix form as

$$\begin{bmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 1 + j1.5 & j2.5 \\ 11 & 15 \end{vmatrix} = 15 - j5$$

$$\Delta_1 = \begin{vmatrix} 20 & j2.5 \\ 0 & 15 \end{vmatrix} = 300, \qquad \Delta_2 = \begin{vmatrix} 1 + j1.5 & 20 \\ 11 & 0 \end{vmatrix} = -220$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{300}{15 - j5} = 18.97 \underline{/18.43^\circ} \text{ V}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-220}{15 - j5} = 13.91 \underline{/198.3^\circ} \text{ V}$$

The current I_x is given by

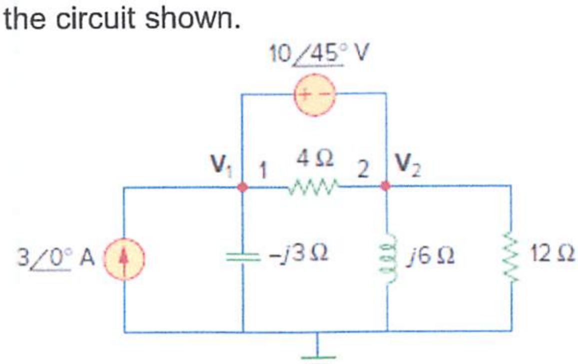
$$I_x = \frac{V_1}{-j2.5} = \frac{18.97 \underline{/18.43^\circ}}{2.5 \underline{/-90^\circ}} = 7.59 \underline{/108.4^\circ} \text{ A}$$

Transforming this to the time domain,

$$i_x = 7.59 \cos(4t + 108.4^\circ) \text{ A}$$

Example 1.2

Compute V_1 and V_2 in the circuit shown.



Solution:

Nodes 1 and 2 form a supernode as shown above. Applying KCL at the supernode gives

$$3 = \frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12}$$

or

$$36 = j4V_1 + (1 - j2)V_2 \tag{1}$$

But a voltage source is connected between nodes 1 and 2, so that

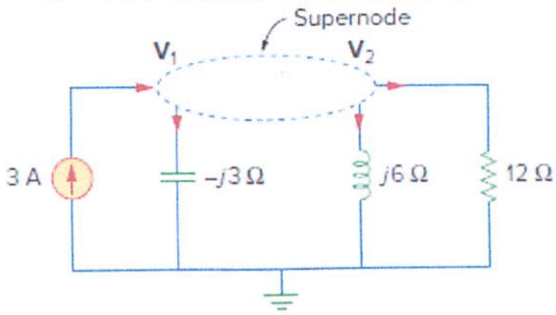
$$V_1 = V_2 + 10 \underline{/45^\circ} \tag{2}$$

Substituting Eq. (2) in Eq. (1) results in

$$36 - 40 \underline{/135^\circ} = (1 + j2)V_2 \qquad \Rightarrow \qquad V_2 = 31.41 \underline{/-87.18^\circ} \text{ V}$$

From Eq. (2),

$$\mathbf{V}_1 = \mathbf{V}_2 + 10\,\underline{/45^\circ} = 25.78\,\underline{/-70.48^\circ}\text{ V}$$

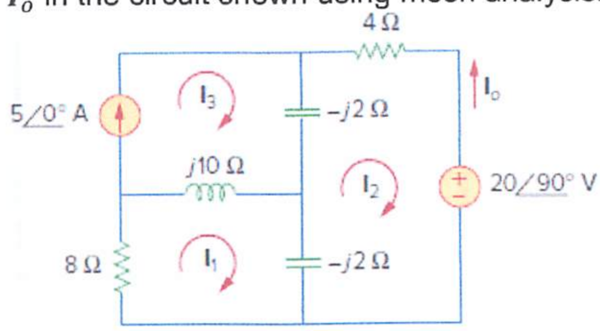


1.1.1 Mesh Analysis

Kirchhoff's voltage law (KVL) forms the basis of mesh analysis. The validity of KVL for ac circuits is illustrated in the following examples.

Example 1.3

Determine the current I_o in the circuit shown using mesh analysis.



Solution:

Applying KVL to mesh 1, we obtain

$$(8 + j10 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - j10\mathbf{I}_3 = 0 \qquad (1)$$

For mesh 2,

$$(4 - j2 - j2)\mathbf{I}_2 - (-j2)\mathbf{I}_1 - (-j2)\mathbf{I}_3 + 20\,\underline{/90^\circ} = 0 \qquad (2)$$

For mesh 3, $\mathbf{I}_3 = 5$. Substituting this in Eqs. (1) and (2), we get

$$(8 + j8)\mathbf{I}_1 + j2\mathbf{I}_2 = j50 \qquad (3)$$

$$j2\mathbf{I}_1 + (4 - j4)\mathbf{I}_2 = -j20 - j10 \qquad (4)$$

Equations (3) and (4) can be put in matrix form as

$$\begin{bmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{bmatrix}\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} j50 \\ -j30 \end{bmatrix}$$

from which we obtain the determinants

$$\Delta = \begin{vmatrix} 8 + j8 & j2 \\ j2 & 4 - j4 \end{vmatrix} = 32(1 + j)(1 - j) + 4 = 68$$

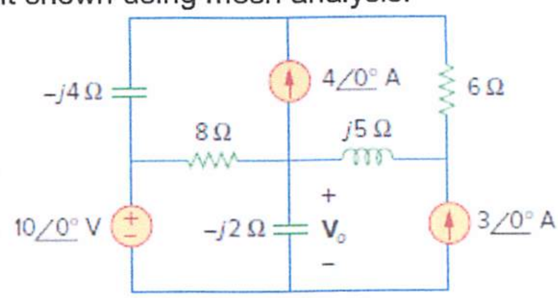
$$\Delta_2 = \begin{vmatrix} 8 + j8 & j50 \\ j2 & -j30 \end{vmatrix} = 340 - j240 = 416.17\,\underline{/-35.22^\circ}$$

$$\mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \frac{416.17\angle{-35.22^\circ}}{68} = 6.12\angle{-35.22^\circ}\text{ A}$$

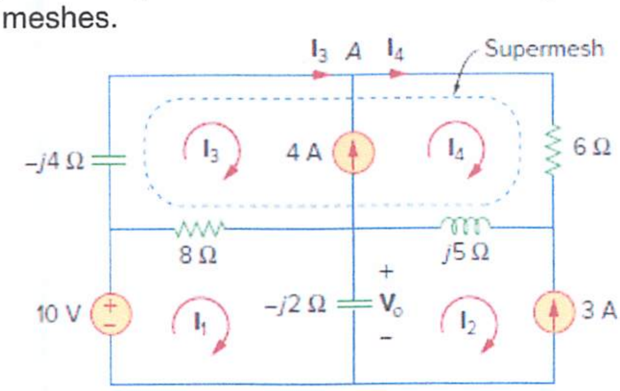
The desired current is

$$\mathbf{I}_o = -\mathbf{I}_2 = 6.12\angle{144.78^\circ}\text{ A}$$

Example 1.4
Solve for $\mathbf{V}_o$ in the circuit shown using mesh analysis.



Solution:
As shown in the figure below, meshes 3 and 4 form a supermesh due to the current source between the meshes.



For mesh 1, KVL gives

$$-10 + (8 - j2)\mathbf{I}_1 - (-j2)\mathbf{I}_2 - 8\mathbf{I}_3 = 0$$

or

$$(8 - j2)\mathbf{I}_1 + j2\mathbf{I}_2 - 8\mathbf{I}_3 = 10 \tag{1}$$

For mesh 2,

$$\mathbf{I}_2 = -3 \tag{2}$$

For the supermesh,

$$(8 - j4)\mathbf{I}_3 - 8\mathbf{I}_1 + (6 + j5)\mathbf{I}_4 - j5\mathbf{I}_2 = 0 \tag{3}$$

Due to the current source between meshes 3 and 4, at node A,

$$\mathbf{I}_4 = \mathbf{I}_3 + 4 \tag{4}$$

Instead of solving the above four equations, we reduce them to two by elimination.
Combining Eqs. (1) and (2),

$$(8 - j2)\mathbf{I}_1 - 8\mathbf{I}_3 = 10 + j6 \tag{5}$$

Combining Eqs. (2) to (4),

$$-8\mathbf{I}_1 + (14 + j)\mathbf{I}_3 = -24 - j35 \tag{6}$$

From Eqs. (5) and (6), we obtain the matrix equation

$$\begin{bmatrix} 8-j2 & -8 \\ -8 & 14+j \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 10+j6 \\ -24-j35 \end{bmatrix}$$

We obtain the following determinants

$$\Delta = \begin{vmatrix} 8-j2 & -8 \\ -8 & 14+j \end{vmatrix} = 112 + j8 - j28 + 2 - 64 = 50 - j20$$

$$\Delta_1 = \begin{vmatrix} 10+j6 & -8 \\ -24-j35 & 14+j \end{vmatrix} = 140 + j10 + j84 - 6 - 192 - j280$$
$$= -58 - j186$$

Current $\mathbf{I}_1$ is obtained as

$$\mathbf{I}_1 = \frac{\Delta_1}{\Delta} = \frac{-58 - j186}{50 - j20} = 3.618 \underline{/274.5^\circ} \text{ A}$$

The required voltage $\mathbf{V}_o$ is

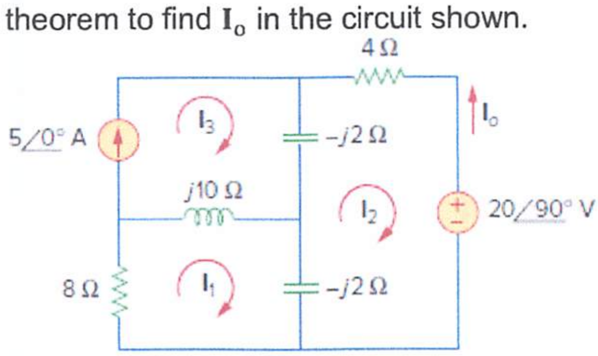
$$\mathbf{V}_o = -j2(\mathbf{I}_1 - \mathbf{I}_2) = -j2(3.618 \underline{/274.5^\circ} + 3)$$
$$= -7.2134 - j6.568 = 9.756 \underline{/222.32^\circ} \text{ V}$$

1.2 SUPERPOSITION THEOREM

Since ac circuits are linear, the superposition theorem applies to ac circuits the same way it applies to dc circuits. When a circuit has sources operating at *different* frequencies, one must add the responses due to the individual frequencies in the time domain.

Example 1.5

Use the superposition theorem to find $\mathbf{I}_o$ in the circuit shown.



Solution:

Let

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o \qquad (1)$$

where $\mathbf{I}'_o$ and $\mathbf{I}''_o$ are due to the voltage and current sources, respectively. To find $\mathbf{I}'_o$, consider the circuit in figure (a). If we let **Z** be the parallel combination of $-j2$ and $8 + j10$, then

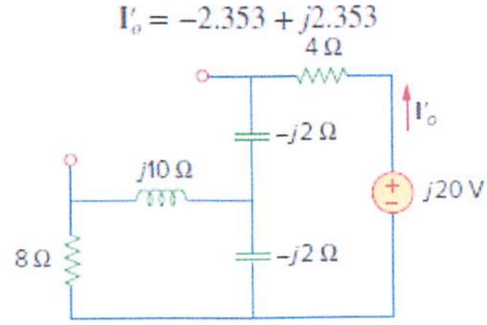
$$\mathbf{Z} = \frac{-j2(8 + j10)}{-2j + 8 + j10} = 0.25 - j2.25$$

and current $\mathbf{I}'_o$ is

$$\mathbf{I}'_o = \frac{j20}{4 - j2 + \mathbf{Z}} = \frac{j20}{4.25 - j4.25}$$

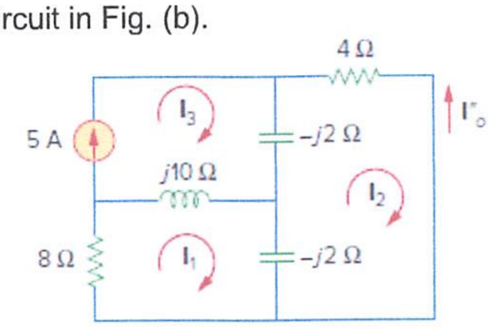
or

$$\mathbf{I}'_o = -2.353 + j2.353 \tag{2}$$



(a)

To get $\mathbf{I}''_o$, consider the circuit in Fig. (b).



(b)

For mesh 1,

$$(8 + j8)\mathbf{I}_1 - j10\mathbf{I}_3 + j2\mathbf{I}_2 = 0 \tag{3}$$

For mesh 2,

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j2\mathbf{I}_3 = 0 \tag{4}$$

For mesh 3,

$$\mathbf{I}_3 = 5 \tag{5}$$

From Eqs. (4) and (5),

$$(4 - j4)\mathbf{I}_2 + j2\mathbf{I}_1 + j10 = 0$$

Expressing $\mathbf{I}_1$ in terms of $\mathbf{I}_2$ gives

$$\mathbf{I}_1 = (2 + j2)\mathbf{I}_2 - 5 \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (3), we get

$$(8 + j8)[(2 + j2)\mathbf{I}_2 - 5] - j50 + j2\mathbf{I}_2 = 0$$

or

$$\mathbf{I}_2 = \frac{90 - j40}{34} = 2.647 - j1.176$$

Current $\mathbf{I}''_o$ is obtained as

$$\mathbf{I}''_o = -\mathbf{I}_2 = -2.647 + j1.176 \tag{7}$$

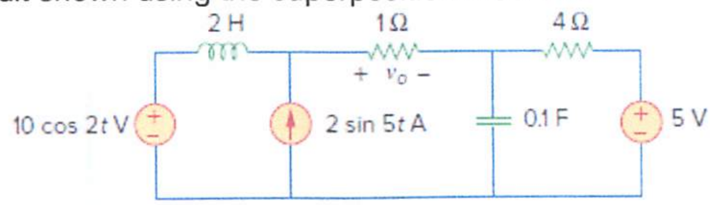
From Eqs. (2) and (7), we write

$$\mathbf{I}_o = \mathbf{I}'_o + \mathbf{I}''_o = -5 + j3.529 = 6.12\underline{/144.78^\circ}\text{ A}$$

which agrees with what we got in Example 1.3. It should be noted that applying the superposition theorem is not the best way to solve this problem. However, in Example 1.6, superposition is clearly the easiest approach.

Example 1.6

Find v_o of the circuit shown using the superposition theorem.



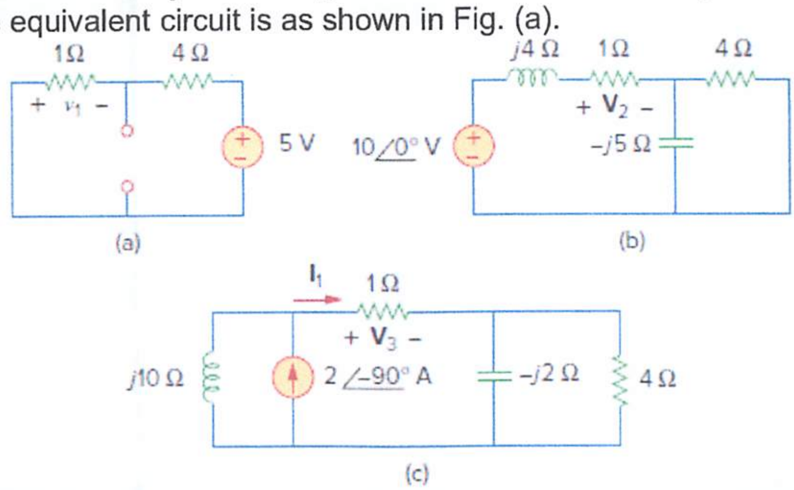
Solution:

Because the circuit operates at three different frequencies ($\omega = 0$ for the dc voltage source), one way to obtain a solution is to use superposition, which breaks the problem into single-frequency problems. So we let

$$v_o = v_1 + v_2 + v_3 \quad (1)$$

where v_1 is due to the 5-V dc voltage source, v_2 is due to the 10 cos 2*t* V voltage source, and v_3 is due to the 2 sin 5*t* A current source.

To find v_1, we set to zero all sources except the 5-V dc source. We recall that at steady state, a capacitor is an open circuit to dc while an inductor is a short circuit to dc. There is an alternative way of looking at this. Because $\omega = 0$, $j\omega L = 0$, $1/j\omega C = \infty$. Either way, the equivalent circuit is as shown in Fig. (a).



Solution of Example 1.6: (a) setting all sources to zero except the 5-V dc source, (b) setting all sources to zero except the ac voltage source, (c) setting all sources to zero except the ac current source.

By voltage division,

$$-v_1 = \frac{1}{1+4}(5) = 1 \text{ V} \quad (2)$$

To find v_2, we set to zero both the 5-V source and the 2 sin 5*t* current source and transform the circuit to the frequency domain.

$$10 \cos 2t \quad \Rightarrow \quad 10\underline{/0^\circ}, \qquad \omega = 2 \text{ rad/s}$$

$$2 \text{ H} \quad \Rightarrow \quad j\omega L = j4\ \Omega$$

$$0.1 \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j5\ \Omega$$

The equivalent circuit is now as shown in Fig. (b). Let

$$\mathbf{Z} = -j5 \parallel 4 = \frac{-j5 \times 4}{4 - j5} = 2.439 - j1.951$$

By voltage division,

$$\mathbf{V}_2 = \frac{1}{1 + j4 + \mathbf{Z}}(10\underline{/0^\circ}) = \frac{10}{3.439 + j2.049} = 2.498\underline{/-30.79^\circ}$$

In the time domain,

$$v_2 = 2.498\cos(2t - 30.79^\circ) \tag{3}$$

To obtain v_3, we set the voltage sources to zero and transform what is left to the frequency domain.

$$2\sin 5t \quad \Rightarrow \quad 2\underline{/-90^\circ}, \quad \omega = 5 \text{ rad/s}$$

$$2 \text{ H} \quad \Rightarrow \quad j\omega L = j10\ \Omega$$

$$0.1 \text{ F} \quad \Rightarrow \quad \frac{1}{j\omega C} = -j2\ \Omega$$

The equivalent circuit is in Fig. (c). Let

$$\mathbf{Z}_1 = -j2 \parallel 4 = \frac{-j2 \times 4}{4 - j2} = 0.8 - j1.6\ \Omega$$

By current division,

$$\mathbf{I}_1 = \frac{j10}{j10 + 1 + \mathbf{Z}_1}(2\underline{/-90^\circ})\text{ A}$$

$$\mathbf{V}_3 = \mathbf{I}_1 \times 1 = \frac{j10}{1.8 + j8.4}(-j2) = 2.328\underline{/-80^\circ}\text{ V}$$

In the time domain,

$$v_3 = 2.33\cos(5t - 80^\circ) = 2.33\sin(5t + 10^\circ)\text{ V} \tag{4}$$

Substituting Eqs. (2) to (4) into Eq. (1), we have

$$v_o(t) = -1 + 2.498\cos(2t - 30.79^\circ) + 2.33\sin(5t + 10^\circ)\text{ V}$$

1.3 SOURCE TRANSFORMATION

Source transformation in the frequency domain involves transforming a voltage source in series with an impedance to a current source in parallel with an impedance, or vice versa.

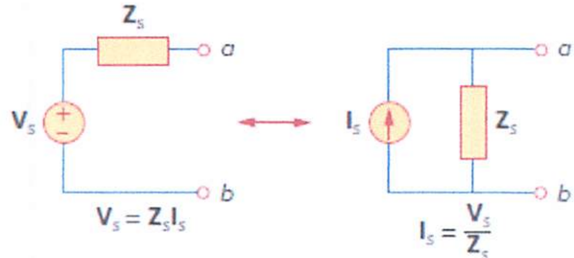
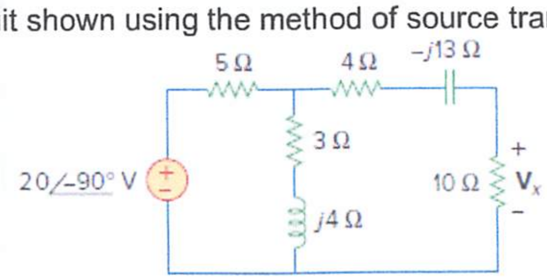


Figure 1.1 Source Transformation

As we go from one source type to another, we must keep the following relationship in mind:

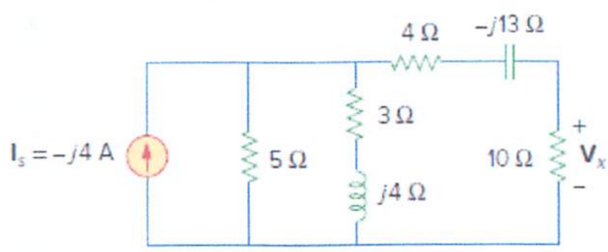
$$\mathbf{V}_s = \mathbf{Z}_s\mathbf{I}_s \quad \Leftrightarrow \quad \mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_s} \tag{1.1}$$

Example 1.7
Calculate $\mathbf{V}_x$ in the circuit shown using the method of source transformation.



Solution:
We transform the voltage source to a current source and obtain the circuit in Fig. (a), where

$$\mathbf{I}_s = \frac{20\angle -90^\circ}{5} = 4\angle -90^\circ = -j4 \text{ A}$$



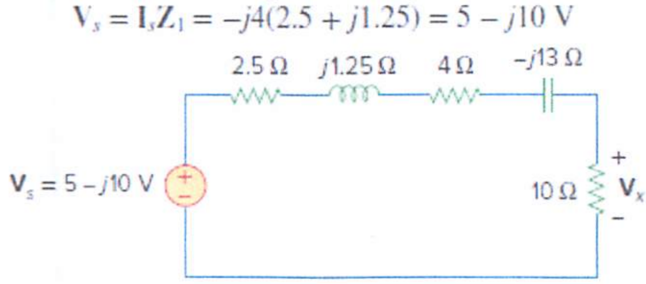
(a)

The parallel combination of 5-Ω resistance and (3 + *j*4) impedance gives

$$\mathbf{Z}_1 = \frac{5(3+j4)}{8+j4} = 2.5 + j1.25\ \Omega$$

Converting the current source to a voltage source yields the circuit in Fig. (b), where

$$\mathbf{V}_s = \mathbf{I}_s\mathbf{Z}_1 = -j4(2.5 + j1.25) = 5 - j10 \text{ V}$$



(b)

By voltage division,

$$\mathbf{V}_x = \frac{10}{10 + 2.5 + j1.25 + 4 - j13}(5 - j10) = 5.519\angle -28^\circ \text{ V}$$

1.4 THEVENIN'S AND NORTON'S THEOREMS

Thevenin's and Norton's theorems are applied to ac circuits in the same way as they are to dc circuits. The frequency domain version of a Thevenin equivalent circuit is depicted in Fig. 1.2, where a linear circuit is replaced by a voltage source in series with an impedance. The Norton equivalent circuit is illustrated in Fig. 1.3, where a linear circuit is replaced by a current source in parallel with an impedance.

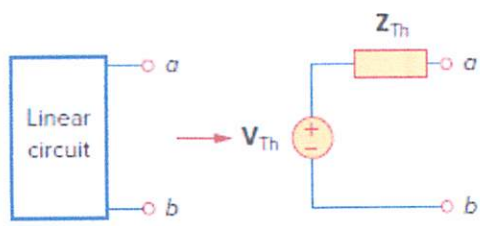


Figure 1.2 Thevenin equivalent.

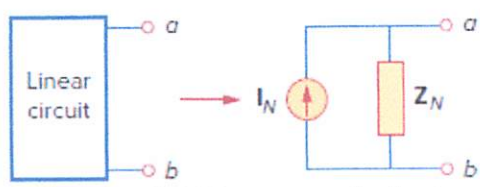


Figure 1.3 Norton equivalent.

Keep in mind that the two equivalent circuits are related as

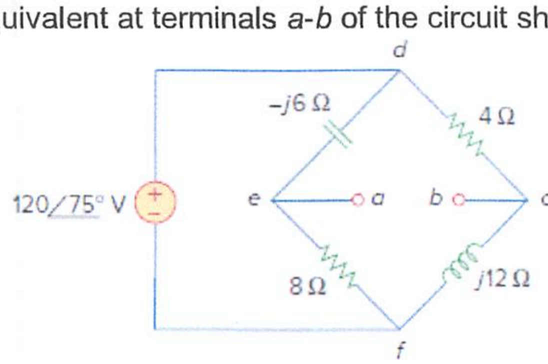
$$\mathbf{V}_{Th} = \mathbf{Z}_N \mathbf{I}_N, \qquad \mathbf{Z}_{Th} = \mathbf{Z}_N \tag{1.2}$$

just as in source transformation. $\mathbf{V}_{Th}$ is the open-circuit voltage while $\mathbf{I}_N$ is the short-circuit current.

If the circuit has sources operating at different frequencies, the Thevenin or Norton equivalent circuit must be determined at each frequency. This leads to entirely different equivalent circuits, one for each frequency, not one equivalent circuit with equivalent sources and equivalent impedances.

Example 1.8

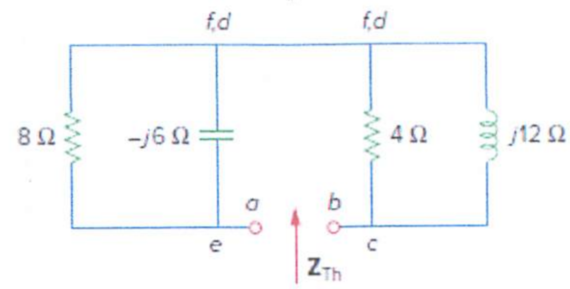
Obtain the Thevenin equivalent at terminals *a-b* of the circuit shown.



Solution:

We find $\mathbf{Z}_{Th}$ by setting the voltage source to zero. As shown below, the 8-Ω resistance is now in parallel with the $-j6$ reactance, so that their combination gives

$$Z_1 = -j6 \parallel 8 = \frac{-j6 \times 8}{8 - j6} = 2.88 - j3.84\ \Omega$$



Similarly, the 4-Ω resistance is in parallel with the $j12$ reactance, and their combination gives

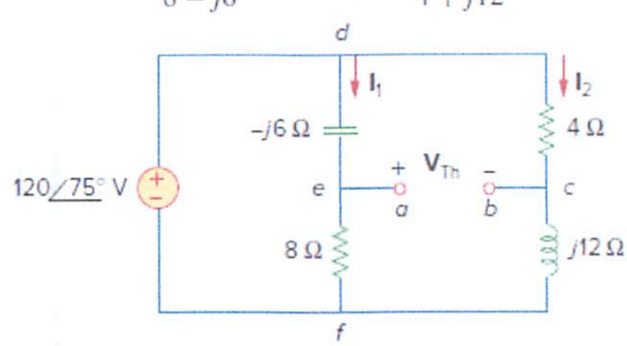
$$Z_2 = 4 \parallel j12 = \frac{j12 \times 4}{4 + j12} = 3.6 + j1.2\ \Omega$$

The Thevenin impedance is the series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$; that is,

$$Z_{Th} = Z_1 + Z_2 = 6.48 - j2.64\ \Omega$$

To find $\mathbf{V}_{Th}$, consider the circuit below. Currents $\mathbf{I}_1$ and $\mathbf{I}_2$ are obtained as

$$\mathbf{I}_1 = \frac{120\angle 75^\circ}{8 - j6}\ \text{A}, \qquad \mathbf{I}_2 = \frac{120\angle 75^\circ}{4 + j12}\ \text{A}$$



Applying KVL around loop *bcdeab* gives

$$\mathbf{V}_{Th} - 4\mathbf{I}_2 + (-j6)\mathbf{I}_1 = 0$$

or

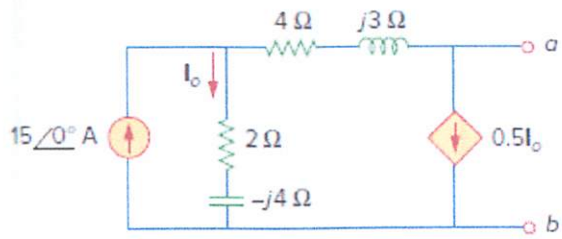
$$\mathbf{V}_{Th} = 4\mathbf{I}_2 + j6\mathbf{I}_1 = \frac{480\angle 75^\circ}{4 + j12} + \frac{720\angle 75^\circ + 90^\circ}{8 - j6}$$

$$= 37.95\angle 3.43^\circ + 72\angle 201.87^\circ$$

$$= -28.936 - j24.55 = 37.95\angle 220.31^\circ\ \text{V}$$

Example 1.9

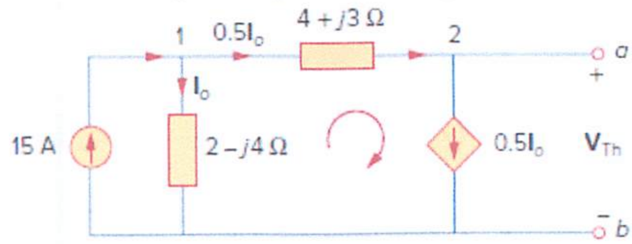
Find the Thevenin equivalent of the circuit as seen from terminals *a-b*.



Solution:

To find $\mathbf{V}_{Th}$, we apply KCL at node 1 in Fig. (a).

$$15 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Rightarrow \quad \mathbf{I}_o = 10 \text{ A}$$



(a)

Applying KVL to the loop on the right-hand side in Fig. (a), we obtain

$$-\mathbf{I}_o(2 - j4) + 0.5\mathbf{I}_o(4 + j3) + \mathbf{V}_{Th} = 0$$

or

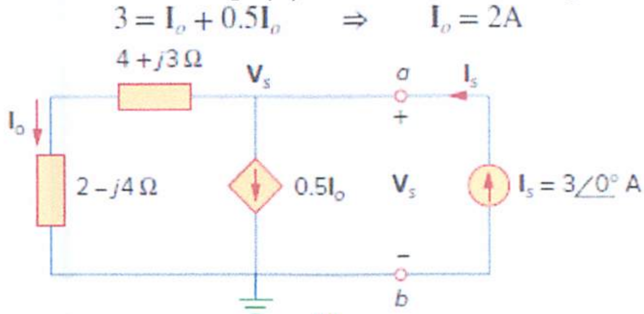
$$\mathbf{V}_{Th} = 10(2 - j4) - 5(4 + j3) = -j55$$

Thus, the Thevenin voltage is

$$\mathbf{V}_{Th} = 55\angle{-90^\circ} \text{ V}$$

To obtain $\mathbf{Z}_{Th}$, we remove the independent source. Due to the presence of the dependent current source, we connect a 3-A current source (3 is an arbitrary value chosen for convenience here, a number divisible by the sum of currents leaving the node) to terminals *a-b* as shown in Fig. (b). At the node, KCL gives

$$3 = \mathbf{I}_o + 0.5\mathbf{I}_o \quad \Rightarrow \quad \mathbf{I}_o = 2\text{A}$$



(b)

Applying KVL to the outer loop in Fig. (b) gives

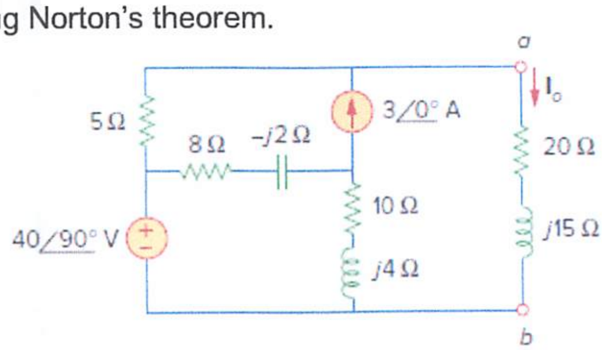
$$\mathbf{V}_s = \mathbf{I}_o(4 + j3 + 2 - j4) = 2(6 - j)$$

The Thevenin impedance is

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_s}{\mathbf{I}_s} = \frac{2(6 - j)}{3} = 4 - j0.6667\ \Omega$$

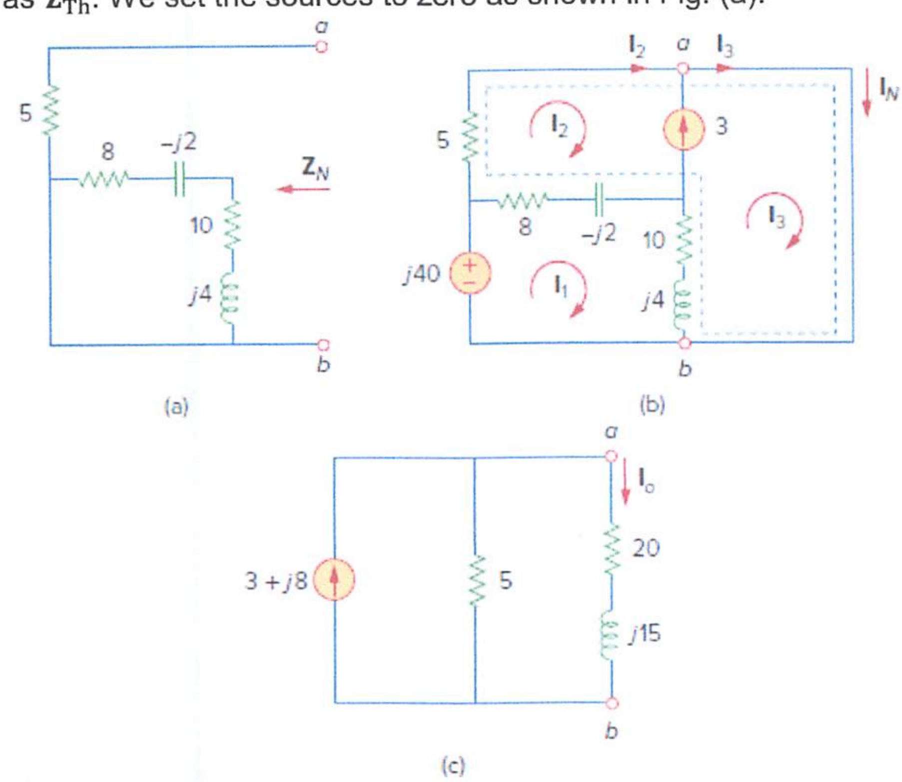
Example 1.10

Obtain current $\mathbf{I}_o$ using Norton's theorem.



Solution:

Our first objective is to find the Norton equivalent at terminals *a-b*. $\mathbf{Z}_N$ is found in the same way as $\mathbf{Z}_{\mathrm{Th}}$. We set the sources to zero as shown in Fig. (a).



As evident from the figure, the (8 − *j*2) and (10 + *j*4) impedances are short-circuited, so that

$$\mathbf{Z}_N = 5\ \Omega$$

To get $\mathbf{I}_N$, we short-circuit terminals *a-b* as in Fig. (b) and apply mesh analysis. Notice that meshes 2 and 3 form a supermesh because of the current source linking them. For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0 \qquad (1)$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0 \qquad (2)$$

At node *a*, due to the current source between meshes 2 and 3,

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 \qquad (3)$$

Adding Eqs. (1) and (2) gives

$$-j40 + 5\mathbf{I}_2 = 0 \qquad \Rightarrow \qquad \mathbf{I}_2 = j8$$

From Eq. (3),

$$\mathbf{I}_3 = \mathbf{I}_2 + 3 = 3 + j8$$

The Norton current is

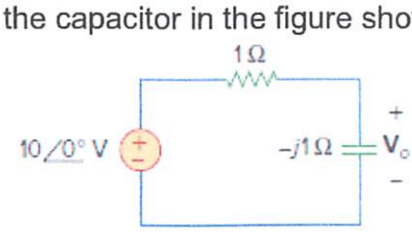
$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\ \text{A}$$

Figure (c) shows the Norton equivalent circuit along with the impedance at terminals *a-b*. By current division,

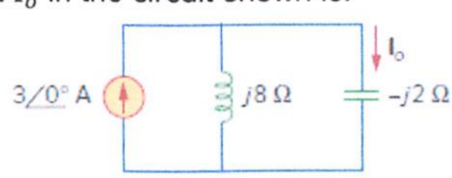
$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\underline{/38.48^\circ}\ \text{A}$$

Self-Evaluation:

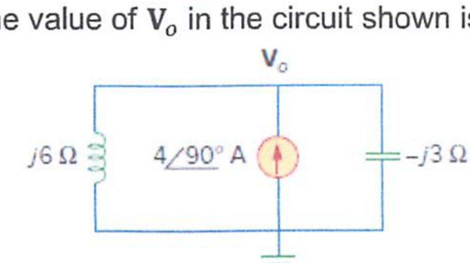
1. The voltage $\mathbf{V}_o$ across the capacitor in the figure shown is:
   a. $5\angle 0°$ V
   b. $7.071\angle 45°$ V
   c. $7.071\angle -45°$ V
   d. $5\angle -45°$ V



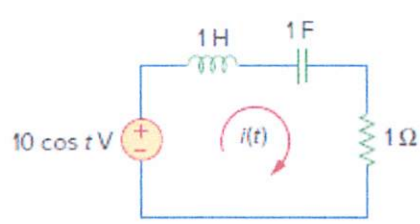
2. The value of the current $\mathbf{I}_o$ in the circuit shown is:
   a. $4\angle 0°$ A
   b. $2.4\angle -90°$ A
   c. $0.6\angle 0°$ A
   d. -1 A



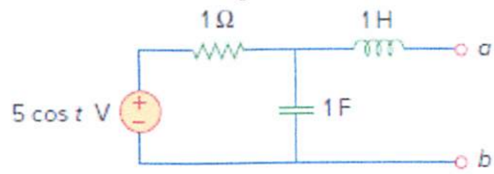
3. Using nodal analysis, the value of $\mathbf{V}_o$ in the circuit shown is:
   a. -24 V
   b. -8 V
   c. 8 V
   d. 24 V



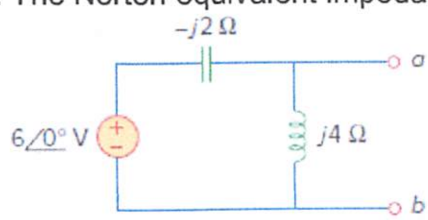
4. In the circuit shown, current *i*(*t*) is:
   a. $10\cos t$ A
   b. $10\sin t$ A
   c. $5\cos t$ A
   d. $10\sin t$ A
   e. $4.472\cos(t - 63.43°)$ A



5. In the circuit shown, the Thevenin voltage at terminals *a*-*b* is:
   a. $3.535\angle -45° \text{ V}$
   b. $3.535\angle 45° \text{ V}$
   c. $7.071\angle -45° \text{ V}$
   d. $7.071\angle 45° \text{ V}$



6. Refer to the circuit below. The Norton equivalent impedance at terminals a-b is:
   a. $-j4\ \Omega$
   b. $-j2\ \Omega$
   c. $j2\ \Omega$
   d. $j4\ \Omega$



Answers: 1c, 2a, 3d, 4a, 5a, 6a

Review of Concepts:

1. We apply nodal and mesh analysis to ac circuits by applying KCL and KVL to the phasor form of the circuits.
2. In solving for the steady-state response of a circuit that has independent sources with different frequencies, each independent source *must* be considered separately. The most natural approach to analyzing such circuits is to apply the superposition theorem. A separate phasor circuit for each frequency *must* be solved independently, and the corresponding response should be obtained in the time domain. The overall response is the sum of the time domain responses of all the individual phasor circuits.
3. The concept of source transformation is also applicable in the frequency domain.
4. The Thevenin equivalent of an ac circuit consists of a voltage source $\mathbf{V}_{\text{Th}}$ in series with the Thevenin impedance $\mathbf{Z}_{\text{Th}}$.
5. The Norton equivalent of an ac circuit consists of a current source $\mathbf{I}_N$ in parallel with the Norton impedance $\mathbf{Z}_{\text{N}}$ $(= \mathbf{Z}_{\text{Th}})$.

References:

- Charles Alexander, Matthew Sadiku-*Fundamentals of Electric Circuits* (McGraw-Hill Education; 6th ed. 2016)
- Mahmood Nahvi, PhD. & Joseph A. Edminister- *Schaum's Outlines of Electric Circuits* (McGraw-Hill Education; 7th ed. 2017)